CHAPTER 13

The Optimal Rate of Inflation☆

Stephanie Schmitt-Grohé* and Martín Uribe**
*Columbia University, CEPR, and NBER
**Columbia University and NBER

Contents

☆ We thank Jordi Galí and Pedro Teles for comments.

Handbook of Monetary Economics, Volume 3B
ISSN 0169-7218, DOI: 10.1016/S0169-7218(11)03019-X

Abstract

Observed inflation targets around the industrial world are concentrated at two percent per year. This chapter investigates the extent to which the observed magnitudes of inflation targets are consistent with the optimal rate of inflation predicted by leading theories of monetary non-neutrality. We find that consistently those theories imply that the optimal rate of inflation ranges from minus the real rate of interest to numbers insignificantly above zero. Furthermore, we argue that the zero bound on nominal interest rates does not represent an impediment for setting inflation targets near or below zero. Finally, we find that central banks should adjust their inflation targets upward by the size of the quality bias in measured inflation only if hedonic prices are more sticky than nonquality-adjusted prices.

JEL classification: E31, E4, E5

Keywords

Downward Nominal Rigidities
Foreign Demand for Money
Friedman Rule
Quality Bias
Ramsey Policy
Sticky-Prices
Zero Bound

1. INTRODUCTION

The inflation objectives of virtually all central banks around the world are significantly above zero. Among monetary authorities in industrial countries that self-classify as inflation targeters, for example, inflation targets are concentrated at a level of 2% per year (Table 1). Inflation objectives are about one percentage point higher in inflation-targeting emerging countries. The central goal of this chapter is to investigate the extent to which the observed magnitudes of inflation targets are consistent with the optimal rate of inflation predicted by leading theories of monetary non-neutrality. We find that consistently those theories imply that the optimal rate of inflation ranges from minus the real rate of interest to numbers insignificantly above zero. Our findings suggest that the empirical regularity regarding the size of inflation targets cannot be reconciled with the optimal long-run inflation rates predicted by existing theories. In this sense, the observed inflation objectives of central banks pose a puzzle for monetary theory.

In the existing literature, two major sources of monetary non-neutrality govern the determination of the optimal long-run rate of inflation. One source is a nominal friction stemming from a demand for fiat money. The second source is given by the assumption of price stickiness.

In monetary models in which the only nominal friction takes the form of a demand for fiat money for transaction purposes, optimal monetary policy calls for minimizing the opportunity cost of holding money by setting the nominal interest rate to zero.

Table 1 Inflation Targets Around the World

Country	Inflation target (%)
Industrial countries	
New Zealand	1–3
Canada	1–3
United Kingdom	2
Australia	2–3
Sweden	2 ± 1
Switzerland	< 2
Iceland	2.5
Norway	2.5
Emerging countries	
Israel	1–3
Czech Republic	3 ± 1
Korea	2.5–3.5
Poland	2.5 ± 1
Brazil	4.5 ± 2.5
Chile	2–4
Colombia	5 ± 1.5
South Africa	3–6
Thailand	0–3.5
Mexico	3 ± 1
Hungary	3.5 ± 1
Peru	2.5 ± 1
Philippines	5–6

Source: World Economic Outlook, 2005. Table 4.1.

This policy, also known as the Friedman rule, implies an optimal rate of inflation that is negative and equal in absolute value to the real rate of interest. If the long-run real rate of interest lies, say, between 2 and 4%, the optimal rate of inflation predicted by this class of models would lie between -2 and -4%. This prediction is clearly at odds with observed inflation targets. A second important result that emerges in this class of models is that the Friedman rule is optimal regardless of whether the government is assumed to finance its budget via lump-sum taxes or via distortionary income taxes. This result has been given considerable attention in the literature because it runs against the conventional wisdom that in a second-best world all goods, including money holdings, should be subject to taxation.

One way to induce optimal policy to deviate from the Friedman rule in this type of model is to assume that the tax system is incomplete. We study three sources of tax incompleteness that give rise to optimal inflation rates above the one consistent with the Friedman rule: untaxed profits due to decreasing returns to scale with perfect competition in product markets, untaxed profits due to monopolistic competition in product markets, and untaxed income due to tax evasion. These three cases have in common that the monetary authority finds it optimal to use inflation as an indirect levy on pure rents that would otherwise remain untaxed. We evaluate these three avenues for rationalizing optimal deviations from the Friedman rule both analytically and quantitatively. We find that in all three cases the share of untaxed income required to justify an optimal inflation rate of about 2%, which would be in line with observed inflation targets, is unreasonably large (above 30%). We conclude that tax incompleteness is an unlikely candidate for explaining the magnitude of actual inflation targets.

Countries whose currency is used abroad may have incentives to deviate from the Friedman rule as a way to collect resources from foreign residents. This rationale for a positive inflation target is potentially important for the United States, the bulk of whose currency circulates abroad. Motivated by these observations, we characterize the optimal rate of inflation in an economy with a foreign demand for its currency in the context of a model in which, in the absence of such foreign demand, the Friedman rule would be optimal. We show analytically that once a foreign demand for domestic currency is taken into account, the Friedman rule ceases to be Ramsey optimal. Calibrated versions of the model that match the range of empirical estimates of the size of foreign demand for U.S. currency deliver Ramsey optimal rates of inflation between 2 and 10% per annum. The fact that developed countries whose currency is hardly demanded abroad, such as Canada, New Zealand, and Australia, set inflation targets similar to those that have been estimated for the United States, suggests that although the United States does have incentives to tax foreign dollar holdings via inflation, it must not be acting on such incentives. The question of why the United States appears to leave this margin unexploited deserves further study.

Overall, our examination of models in which a transactional demand for money is the sole source of nominal friction leads us to conclude that this class of models fails to provide a compelling explanation for the magnitude of observed inflation targets.

The second major source of monetary non-neutrality studied in the literature is given by nominal rigidities in the form of sluggish price adjustment. Models that incorporate this type of friction as the sole source of monetary non-neutrality predict that the optimal rate of inflation is zero. This prediction of the sticky-price model is robust in assuming that nominal prices are partially indexed to past inflation. The reason for the optimality of price stability is that it eliminates the inefficiencies brought about by the presence of price-adjustment costs. Clearly, the sticky-price friction brings the optimal rate of inflation much closer to observed inflation targets than does the money-demand friction. However, the predictions of the sticky-price model for the optimal rate of inflation still fall short of the 2% inflation target prevailing in developed economies and the 3% inflation target prevailing in developing countries.

One might be led to believe that the problem of explaining observed inflation targets is more difficult than the predictions of the sticky-price model suggest. For a realistic model of the monetary transmission mechanism, it must incorporate both major sources of monetary non-neutrality, price stickiness, and a transactional demand for fiat money. Indeed, in such a model the optimal rate of inflation falls in between the one called for by the money demand friction — deflation at the real rate of interest — and the one called for by the sticky-price friction — zero inflation. The intuition behind this result is straightforward. The benevolent government faces a trade-off between minimizing price adjustment costs and minimizing the opportunity cost of holding money. Quantitative analysis of this trade-off, however, suggests that under plausible model parameterizations, it is resolved in favor of price stability.

The theoretical arguments considered thus far leave the predicted optimal inflation target at least two percentage points below its empirical counterpart. We therefore consider three additional arguments that have been proposed as possible explanations of this gap: the zero bound on nominal interest rates, downward nominal rigidities in factor prices, and a quality bias in the measurement of inflation.

It is often argued in policy circles that at zero or negative rates of inflation the risk of hitting the zero lower bound on nominal interest rates would severely restrict the central bank's ability to conduct successful stabilization policy. The validity of this argument depends critically on the predicted volatility of the nominal interest rate under the optimal monetary policy regime. To investigate the plausibility of this explanation of positive inflation targets, we characterize optimal monetary policy in the context of a medium-scale macroeconomic model estimated to fit business cycles in post-war United States. We find that under the optimal monetary policy the inflation rate has a mean of -0.4%. More important, the optimal nominal interest rate has a mean of 4.4% and a standard deviation of 0.9%. This finding implies that hitting the zero bound would require a decline in the equilibrium nominal interest rate of more than four standard deviations. We regard such an event as highly unlikely. This statement should not to be misinterpreted as meaning that given an inflation target of -0.4% the economy would face a negligible chance of hitting the zero bound under any monetary policy. The correct interpretation is more narrow; namely that such event would be improbable under the optimal policy regime.

The second additional rationale for targeting positive inflation that we address is the presence of downward nominal rigidities. When nominal prices are downwardly rigid, then any relative price change must be associated with an increase in the nominal price level. It follows that to the extent that over the business cycle variations in relative prices are efficient, a positive rate of inflation, aimed at accommodating such changes may be welfare improving. Perhaps the most prominent example of a downwardly rigid price is the nominal wage. A natural question, therefore, is how much inflation is necessary to "grease the wheel of the labor market." The answer appears to be not much. An incipient literature using estimated macroeconomic models with downwardly rigid nominal wages finds optimal rates of inflation below 50 basis points.

The final argument for setting inflation targets significantly above zero that we consider is the well-known fact that due to unmeasured quality improvements in consumption goods the consumer price index overstates the true rate of inflation. For example, in the United States a Senate appointed commission of prominent academic economists established that in the year 1995–1996 the quality bias in CPI inflation was about 0.6% per year. We therefore analyze whether the central bank should adjust its inflation target to account for the systematic upward bias in measured inflation. We show that the answer to this question depends crucially on what prices are assumed to be sticky. Specifically, if non-quality-adjusted prices are sticky, then the inflation target should not be corrected. If, on the other hand, quality-adjusted (or hedonic) prices are sticky, then the inflation target should be raised by the magnitude of the bias. Ultimately, it is an empirical question whether nonquality adjusted or hedonic prices are more sticky. This question is yet to be addressed by the empirical literature on price rigidities.

Throughout this chapter, we refer to the optimal rate of inflation as the one that maximizes the welfare of the representative consumer. We limit attention to Ramsey optimality; that is, the government is assumed to be able to commit to its policy announcements. Finally, in all of the models considered, households and firms are assumed to be optimizing agents with rational expectations.

2. MONEY DEMAND AND THE OPTIMAL RATE OF INFLATION

When the central nominal friction in the economy originates in the need of economic agents to use money to perform transactions, under quite general conditions, optimal monetary policy calls for a zero opportunity cost of holding money. This result is known as the Friedman rule. In fiat money economies in which assets used for transactions purposes do not earn interest, the opportunity cost of holding money equals the nominal interest rate. Therefore, in the class of models in which the demand for money is the central nominal friction, the optimal monetary policy prescribes that the risk-less nominal interest rate — for example, the return on Federal funds — be set at zero at all times. Because in the long run inflationary expectations are linked to the differential between nominal and real rates of interest, the Friedman rule ultimately leads to deflation at the real rate of interest.

A money demand friction can be motivated in a variety of ways, including a cash-in-advance constraint (Lucas, 1982), money in the utility function (Sidrauski, 1967), a shopping-time technology (Kimbrough, 1986), or a transactions-cost technology (Feenstra, 1986). Regardless of how a demand for money is introduced, the intuition for why the Friedman rule is optimal when the single nominal friction stems from the demand for money is straightforward: real money balances provide valuable transaction services to households and firms. At the same time, the cost of printing money is negligible. Therefore, it is efficient to set the opportunity cost of holding money, given by the nominal interest rate, as low as possible. A further reason why the Friedman rule is optimal is that a positive interest rate can distort the efficient allocation of resources. For instance, in the cash-in-advance model with cash and credit goods, a positive interest rate distorts the allocation of private spending across these two types of goods. In models in which money ameliorates transaction costs or decreases shopping time, a positive interest rate introduces a wedge in the consumption-leisure choice.

To illustrate the optimality of the Friedman rule, consider augmenting a neoclassical model with a transaction cost that is decreasing in real money holdings and increasing in consumption spending. Specifically, consider an economy populated by a large number of identical households. Each household has preferences defined over sequences of consumption and leisure and described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t,\ h_t), \tag{1}$$

where c_t denotes consumption, h_t denotes labor effort, and $\beta \in (0,1)$ denotes the subjective discount factor. The single period utility function U is assumed to be increasing in consumption, decreasing in effort, and strictly concave.

A demand for real balances is introduced into the model by assuming that nominal money holdings, denoted M_t, facilitate consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost $s(v_t)$ that is decreasing in the household's money-to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t}, \tag{2}$$

where P_t denotes the nominal price of the consumption good in period t. The transaction cost function, $s(v)$, satisfies the following assumptions: (a) $s(v)$ is non-negative and twice continuously differentiable; (b) there exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (c) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$; and (d) $2s'(v) + vs''$;$(v) > 0$ for all $v \geq \underline{v}$. Assumption (b) ensures that the Friedman rule, that is, a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption (c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level. Assumption (d) ensures that the demand for money is a decreasing function of the nominal interest rate.

Households are assumed to have access to one-period nominal bonds, denoted B_t, which carry a gross nominal interest rate of R_t when held from period t to period $t + 1$. Households supply labor services to competitive labor markets at the real wage rate w_t. In addition, households receive profit income in the amount Π_t from the ownership of firms. The flow budget constraint of the household in period t is then given by:

$$P_t c_t[1 + s(v_t)] + P_t\tau_t + M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_t(w_t h_t + \Pi_t), \tag{3}$$

where τ_t denotes real taxes paid in period t. In addition, it is assumed that the household is subject to the following borrowing limit that prevents it from engaging in Ponzi-type schemes:

$$\lim_{j\to\infty} \frac{M_{t+j} + R_{t+j}B_{t+j}}{\Pi_{s=0}^{j} R_{t+s}} \geq 0. \tag{4}$$

This restriction states that in the long run the household's net nominal liabilities must grow at a rate smaller than the nominal interest rate. It rules out, for example, schemes in which households roll over their net debts forever.

The household chooses sequences $\{c_t, h_t, v_t, M_t, B_t\}_{t=0}^{\infty}$ to maximize Eq. (1) subject to Eqs. (2)–(4), taking as given the sequences $\{P_t, \tau_t, R_t, w_t, \Pi_t\}_{t=0}^{\infty}$ and the initial condition $M_{-1} + R_{-1}B_{-1}$. The first-order conditions associated with the household's maximization problem are Eqs. (2)–(4) holding with equality, and

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t} \tag{5}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{w_t}{1 + s(v_t) + v_t s'(v_t)} \tag{6}$$

$$\frac{U_c(c_t, h_t)}{1 + s(v_t) + v_t s'(v_t)} = \beta \frac{R_t}{\pi_{t+1}} \frac{U_c(c_{t+1}, h_{t+1})}{[1 + s(v_{t+1}) + v_{t+1}s'(v_{t+1})]}, \tag{7}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of price inflation in period t. Optimality condition (5) can be interpreted as a demand for money or liquidity preference function. Given our maintained assumptions about the transactions technology $s(v_t)$, the implied money demand function is decreasing in the gross nominal interest rate R_t. Further, our assumptions imply that as the interest rate vanishes, or R_t approaches unity, the demand for money reaches a finite maximum level given by $C_t/\underline{v}$. At this level of money demand, households are able to perform transactions costlessly, as the transactions cost, $s(v_t)$, becomes zero. Optimality condition (6) shows that a level of money velocity above the satiation level $\underline{v}$, or, equivalently, an interest rate greater than zero, introduces a wedge, given by $1 + s(v_t) + v_t s'(v_t)$, between the marginal rate of substitution of consumption for leisure and the real wage rate. This wedge induces households to move to an inefficient allocation featuring too much leisure and too little consumption. The wedge is increasing in the nominal interest rate, implying that the larger the nominal interest rate, the more distorted is the consumption-leisure choice. Optimality condition (7) is a Fisher equation stating that the nominal interest rate must be equal to the sum of the expected rate of inflation and the real rate of interest. It is clear from the Fisher equation that intertemporal movements in the nominal interest rate create a distortion in the real interest rate perceived by households.

Final goods are produced by competitive firms using the technology $F(h_t)$ that takes labor as the only factor input. The production function F is assumed to be increasing and concave. Firms choose labor input to maximize profits, which are given by

$$\Pi_t = F(h_t) - w_t h_t. \tag{8}$$

The first-order condition associated with the firm's profit maximization problem gives rise to the following demand for labor

$$F'(h_t) = w_t.$$

The government prints money, issues nominal, one-period bonds, and levies taxes to finance an exogenous stream of public consumption, denoted g_t and interest obligations on the outstanding public debt. Accordingly, the government's sequential budget constraint is given by

$$B_t + M_t + P_t \tau_t = R_{t-1} B_{t-1} + M_{t-1} + P_t g_t.$$

In this section, the government is assumed to follow a fiscal policy where taxes are lump sum and government spending and public debt are zero at all times. In addition, the initial amount of public debt outstanding, B_{-1}, is assumed to be zero. These assumptions imply that the government budget constraint simplifies to

$$P_t \tau_t^L + M_t - M_{t-1} = 0,$$

where τ_t^L denotes real lump-sum taxes. According to this expression, the government rebates all seignorage income to households in a lump-sum fashion.

A competitive equilibrium is a set of sequences $\{c_t, h_t, v_t\}$ satisfying Eq. (5) and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{F'(h_t)}{1 + s(v_t) + v_t s'(v_t)} \tag{9}$$

$$[1 + s(v_t)]c_t = F(h_t), \tag{10}$$

$$R_t \geq 1, \tag{11}$$

$$\lim_{j \to \infty} \beta^j \frac{U_c(c_{t+j}, h_{t+j})}{1 + s(v_{t+j}) + v_{t+j} s'(v_{t+j})} \frac{c_{t+j}}{v_{t+j}} = 0, \tag{12}$$

given some monetary policy. Equilibrium condition (9) states that the monetary friction places a wedge between the supply of labor and the demand for labor. Equilibrium condition (10) states that a positive interest rate entails a resource loss in the amount of $s(v_t)c_t$. This resource loss is increasing in the interest rate and vanishes only when the nominal interest rate equals zero. Equilibrium condition (11) imposes a zero lower bound on the nominal interest rate. Such a bound is required to prevent the possibility of unbounded arbitrage profits created by taking short positions in nominal bonds and long positions in nominal fiat money, which would result in ill-defined demands for consumption goods by households. Equilibrium condition (12) results from combining the no-Ponzi-game constraint (4) holding with equality with Eqs. (2) and (7).

2.1 Optimality of the Friedman rule with lump-sum taxation

We wish to characterize optimal monetary policy under the assumption that the government has the ability to commit to policy announcements. This policy optimality concept is known as Ramsey optimality. In the context of the present model, the Ramsey optimal monetary policy problem consists of choosing the path of the nominal interest rate that is associated with the competitive equilibrium that yields the highest level of welfare to households. Formally, the Ramsey problem consists of choosing sequences R_t, c_t, h_t, and v_t, to maximize the household's utility function given in Eq. (1) subject to Eqs. (5) and (9)–(12).

As a preliminary step, before addressing the optimality of the Friedman rule, let us consider whether the Friedman rule, that is,

$$R_t = 1, \forall t$$

can be supported as a competitive equilibrium outcome. This task involves finding sequences c_t, h_t, and v_t that, together with $R_t = 1$, satisfy the equilibrium conditions (5) and (9)–(12). Clearly, Eq. (11) is satisfied by the sequence $R_t = 1$. Equation (5) and the assumptions made about the transactions cost function $s(v)$ imply that when R_t equals unity, money velocity is at the satiation level,

$$v_t = \underline{v}\,.$$

This result implies that when the Friedman rule holds the transactions cost $s(v_t)$ vanishes. Then Eqs. (9) and (10) simplify to the two static equations:[1]

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = F'(h_t)$$

and

$$c_t = F(h_t),$$

which jointly determine constant equilibrium levels of consumption and hours. Finally, because the levels of velocity, consumption, and hours are constant over time, and because the subjective discount factor is less than unity, the transversality condition (12) is also satisfied. We have therefore established that there exists a competitive equilibrium in which the Friedman rule holds at all times.

[1] Sufficient, but not necessary, conditions for a unique, positive solution of these two equations are that $-U_h(c, h)/U_c(c, h)$ is positive and increasing in c and h and that $F(h)$ is positive, strictly increasing and that it satisfy the Inada conditions.

Next, we show that this competitive equilibrium is indeed Ramsey optimal. To see this, consider the solution to the social planner's problem

$$\max_{\{c_t,h_t,v_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

subject to the feasibility constraint (10), which we repeat here for convenience:

$$[1 + s(v_t)]\, c_t = F(h_t).$$

The reason this social planner's problem is of interest for establishing the optimality of the Friedman rule is that its solution must deliver a level of welfare that is at least as high as the level of welfare associated with the Ramsey optimal allocation. This is because both the social planner's problem and the Ramsey problem share the objective function (1) and the feasibility constraint (10), but the Ramsey problem is subject to four additional constraints, namely Eqs. (5), (9), (11), and (12). Consider first the social planner's choice of money velocity, v_t. Money velocity enters only in the feasibility constraint but not in the planner's objective function. Because the transaction cost function $s(v)$ has a global minimum at $\underline{v}$, the social planner will set $v_t = \underline{v}$. At the satiation level of velocity $\underline{v}$ the transaction cost vanishes, so it follows that the feasibility constraint simplifies to $c_t = F(h_t)$. The optimal choice of the pair (c_t, h_t) is then given by the solution to $c_t = F(h_t)$ and $-U_h(c_t, h_t)/U_c(c_t, h_t) = F'(h_t)$. But this real allocation is precisely the one associated with the competitive equilibrium in which the Friedman rule holds at all times. We have therefore established that the Friedman rule is Ramsey optimal.

An important consequence of optimal monetary policy in the context of the present model is that prices are expected to decline over time. In effect, by Eq. (7) and taking into account that in the Ramsey equilibrium consumption and leisure are constant over time, expected inflation is given by $\pi_{t+1} = \beta < 1$, for all $t > 0$. Existing macroeconomic models of the business cycle typically assign a value to the subjective discount factor of around 0.96 per annum. Under this calibration, the present model would imply that the average optimal rate of inflation is -4% per year.

It is important to highlight that the Friedman rule has fiscal consequences and requires coordination between the monetary and fiscal authorities. In effect, an implication of the Friedman rule is that nominal money balances shrink at the same rate as prices. The policy authority finances this continuous shrinkage of the money supply by levying lump-sum taxes on households each period. In the present model, the amount of taxes necessary to cover the seignorage losses created by the Friedman rule is given by $\tau^L_t = (1/\beta - 1)\,(M_t/P_t)$.[2] For instance, under a real interest rate of 4% $(1/\beta - 1) = 0.04$, and a level of real balances of

[2] In a growing economy the Friedman rule is associated with deflation as long as the real interest rate is positive (just as in the nongrowing economy) and with seignorage losses as long as the real interest rate exceeds the growth rate, which is the case of greatest interest. For example, with CRRA preferences, the gross real interest rate, r, would equal g^{σ}/β, the inflation rate would equal $1/r$, and seignorage losses would equal $[r/g - 1](M_t/P_t)$, where g is the growth rate of output and σ is the inverse of the intertemporal elasticity of substitution.

20% of GDP, the required level of taxes would be about 0.8% of GDP. The fiscal authority would have to transfer this amount of resources to the central bank each year in order for the latter to be able to absorb the amount of nominal money balances necessary to keep the money supply at the desired level. Suppose the fiscal authority was unwilling to subsidize the central bank in this fashion. Then the optimal-monetary-policy problem would be like the one discussed thus far, but with the additional constraint that the growth rate of the nominal money supply cannot be negative, $M_t \geq M_{t-1}$. This restriction would force the central bank to deviate from the Friedman rule, potentially in significant ways. For instance, if in the deterministic model discussed thus far one restricts attention to equilibria in which the nominal interest rate is constant and preferences are log-linear in consumption and leisure, then the restricted Ramsey policy would call for price stability, $P_t = P_{t-1}$, and a positive interest rate equal to the real rate of interest, $R_t = 1/\beta$.

The optimality of negative inflation at a rate close to the real rate of interest is robust to adopting any of the alternative motives for holding money discussed at the beginning of this section. It is also robust to the introduction of uncertainty in various forms, including stochastic variations in total factor productivity, preference shocks, and government spending shocks. However, the desirability of sizable average deflation is at odds with the inflation objective of virtually every central bank. It follows that the money demand friction must not be the main factor shaping policymakers' views regarding the optimal level of inflation. For this reason, we now turn to analyzing alternative theories of the cost and benefits of price inflation.

3. MONEY DEMAND, FISCAL POLICY AND THE OPTIMAL RATE OF INFLATION

Thus far, we have studied an economy in which the fiscal authority has access to lump-sum taxes. In this section, we drop the assumption of lump-sum taxation and replace it with the, perhaps more realistic, assumption of distortionary income taxation. In this environment, the policymaker potentially faces a trade-off between using regular taxes and printing money to finance public outlays. In a provoking paper, Phelps (1973) suggested that when the government does not have access to lump-sum taxes but only to distortionary tax instruments, then the inflation tax should also be used as part of an optimal taxation scheme. The central result reviewed in this section is that, contrary to Phelps' conjecture, the optimality of negative inflation is unaltered by the introduction of public spending and distortionary income taxation.

The optimality of the Friedman rule (and thus of negative inflation) in the context of an optimal fiscal and monetary policy problem has been intensively studied. It was derived by Kimbrough (1986), Guidotti and Végh (1993), and Correia and Teles (1996, 1999) in a shopping time economy; by Chari, Christiano, and Kehoe (1991) in a model with a cash-in-advance constraint; by Chari, Christiano, and Kehoe (1996) in a money-in-the-utility function model; and by Schmitt-Grohé and Uribe (2004b) in a model with a consumption-based transactions cost technology like the one considered here.

The setup of this section deviates from the one considered in the previous section in three dimensions: First, the government no longer has access to lump-sum taxes. Instead, we assume that taxes are proportional to labor income. Formally,

$$\tau_t = \tau_t^h w_t \, h_t,$$

where τ_t^h denotes the labor income tax rate. With this type of distortionary tax, the labor supply Eq. (6) changes to

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{\left(1-\tau_t^h\right) w_t}{1+s(v_t)+v_t s'(v_t)}. \tag{13}$$

According to this expression, increases in the labor income tax rate and in velocity distort the labor supply decision of households in the same way, by inducing them to demand more leisure and less consumption.

A second departure from the model presented in the previous section is that government purchases are positive. Specifically, we assume that the government faces an exogenous sequence of public spending $\{g_t\}_{t=0}^{\infty}$. As a result, the aggregate resource constraint becomes

$$[1+s(v_t)]\, c_t + g_t = F(h_t). \tag{14}$$

Implicit in this specification is the assumption that the government's consumption transactions are not subject to a monetary friction like the one imposed on private purchases of goods. Finally, unlike the model in the previous section, we now assume that public debt is not restricted to zero at all times. The government's sequential budget constraint now takes the form

$$M_t + B_t = M_{t-1} + R_{t-1}\, B_{t-1} + P_t g_t - P_t\, \tau_t^h w_t h_t. \tag{15}$$

A competitive equilibrium is a set of sequences $\{v_t,\ c_t,\ h_t,\ M_t,\ B_t,\ P_t\}_{t=0}^{\infty}$ satisfying Eq. (2); Eq. (4) holding with equality; Eqs. (5), (7), (8), and (11); and Eqs. (13)–(15), given policies $\left\{R_t,\ \tau_t^h\right\}_{t=0}^{\infty}$, the exogenous process $\{g_t\}_{t=0}^{\infty}$, and the initial condition $M_{-1} + R_{-1}\, B_{-1}$.

As in the previous section, our primary goal is to characterize the Ramsey optimal rate of inflation. To this end, we begin by deriving the primal form of the competitive equilibrium. Then we state the Ramsey problem. And finally we characterize optimal fiscal and monetary policy.

3.1 The primal form of the competitive equilibrium

Following a long-standing tradition in public finance, we study optimal policy using the primal-form representation of the competitive equilibrium. Finding the primal form involves the elimination of all prices and tax rates from the equilibrium conditions, so that the resulting reduced form involves only real variables. In our economy, the real variables that appear in the primal form are consumption, hours, and money velocity. The primal form of the equilibrium conditions consists of two equations. One equation is a feasibility constraint, given by the resource constraint (14), which must hold at every date. The other equation is a single, present-value constraint known as the implementability constraint. The implementability constraint guarantees that at the prices and quantities associated with every possible competitive equilibrium, the present discounted value of consolidated government surpluses equals the government's total initial liabilities.

Formally, sequences $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ satisfying the feasibility condition (14), which we repeat here for convenience,

$$[1 + s(v_t)]\, c_t + g_t = F(h_t),$$

and the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t,\ h_t) c_t + U_h(c_t,\ h_t)\, h_t + \frac{U_c(c_t,\ h_t)[F'(h_t)h_t - F(h_t)]}{1 + s(v_t) + v_t s'(v_t)} \right\} = \frac{U_c(c_0,\ h_0)}{1 + s(v_0) + v_0 s'(v_0)} \frac{R_{-1}B_{-1} + M_{-1}}{P_0} \tag{16}$$

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

given $(R_{-1}\, B_{-1} + M_{-1})$ and P_0, are the same as those satisfying the set of equilibrium conditions (2); Eq. (4) holding with equality; Eqs. (5), (7), (8), and (11); and Eqs. (13)–(15). In the Appendix at the end of the chapter the proof of this statement is presented in Section 1.

3.2 Optimality of the Friedman rule with distortionary taxation

The Ramsey problem consists of choosing a set of strictly positive sequences $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ to maximize the utility function (1) subject to Eqs. (14), (16), $v_t \geq v$, and $v_t^2 s'(v_t) < 1$, given $R_{-1}\, B_{-1} + M_{-1} > 0$ and P_0. We fix the initial price level arbitrarily to keep the Ramsey planner from engineering a large unexpected initial inflation aimed at reducing the real value of predetermined nominal government liabilities. This assumption is regularly maintained in the literature on optimal monetary and fiscal policy.

We now establish that the Friedman rule is optimal (and hence the optimal rate of inflation is negative) under the assumption that the production technology is linear in hours; that is, $F(h_t) = Ah_t$, where $A > 0$ is a parameter. In this case, wage payments exhaust output and firms make zero profits. This is the case typically studied in the related literature (e.g., Chari et al., 1991). With linear production, the implementability constraint (16) becomes independent of money velocity, v_t, for all $t > 0$. Our strategy to characterize optimal monetary policy is to consider first the solution to a less constrained problem that ignores the requirement $v_t^2 s'(v_t) < 1$, and then to verify that the obtained solution indeed satisfies this requirement. Accordingly, letting

ψ_t denote the Lagrange multiplier on the feasibility constraint (14), the first-order condition of the (less constrained) Ramsey problem with respect to v_t for any $t > 0$ is

$$\psi_t c_t s'(v_t)\,(v_t - \underline{v}) = 0; \quad v_t \geq \underline{v}; \quad \psi_t c_t s'(v_t) \geq 0. \tag{17}$$

Recalling that, by our maintained assumptions regarding the form of the transactions cost technology, $s'(v)$ vanishes at $v = \underline{v}$, it follows immediately that $v_t = \underline{v}$ solves this optimality condition. The omitted constraint $v_t^2 s'(v_t) < 1$ is also clearly satisfied at $v_t = \underline{v}$, since $s'(\underline{v}) = 0$.

From the liquidity preference function (5), it then follows that $R_t = 1$ for all dates $t > 0$.

Finally, because the Ramsey optimality conditions are static and because our economy is deterministic, the Ramsey-optimal sequences of consumption and hours are constant. It then follows from the Fisher equation (7) that the inflation rate $\pi_t - 1$ is negative and equal to $\beta - 1$ for all $t > 1$.

Taking stock, in this section we set out to study the robustness of the optimality of negative inflation to the introduction of a fiscal motive for inflationary finance. We did so by assuming that the government must finance an exogenous stream of government spending with distortionary taxes. The main result of this section is that, in contrast to Phelps's conjecture, negative inflation emerges as optimal even in an environment in which the only source of revenue available to the government, other than seignorage revenue, is distortionary income taxation. Remarkably, the optimality of the Friedman rule obtains independently of the financing needs of the government, embodied in the size of government spending, g_t, and of initial liabilities of the government, $(R_{-1}\,B_{-1} + M_{-1})/P_0$.

A key characteristic of the economic environment studied here that is responsible for the finding that an inflation tax is suboptimal is the absence of untaxed income. In the present framework, with linear production and perfect competition, a labor income tax is equivalent to a tax on the entire GDP. The next section shows, by means of three examples, that when income taxation is incomplete in the sense that it fails to apply uniformly to all sources of income, positive inflation may become optimal as a way to partially restore complete taxation.

4. FAILURE OF THE FRIEDMAN RULE DUE TO UNTAXED INCOME: THREE EXAMPLES

When the government is unable to optimally tax all sources of income, positive inflation may be a desirable instrument to tax the part of income that is suboptimally taxed. The reason is that because at some point all types of private income are devoted to consumption, and because inflation acts as a tax on consumption, a positive nominal interest rate represents an indirect way to tax all sources of income. We illustrate this principle by means of three examples. In two examples firms make pure profits. In one case, pure profits emerge because of decreasing returns to scale in production, and in the other case they are the result of imperfect competition in product markets. In both cases, there is incomplete taxation because the government cannot tax profits at the optimal rate. In the third example, untaxed income stems from tax evasion. In this case, a deviation from the Friedman rule emerges as optimal because, unlike regular taxes, the inflation tax cannot be evaded.

4.1 Decreasing returns to scale

In the model analyzed thus far, suppose that the production technology $F(h)$ exhibits decreasing returns to scale, that is, $F''(h) < 0$. In this case, the first-order condition of the Ramsey problem with respect to v_t for any $t > 0$ is given by

$$\mu_t(v_t - \underline{v}) = 0; \quad v_t \geq \underline{v}; \quad \mu_t \geq 0; \quad \xi_t(1 - v_t^2 s'(v_t)) = 0; \quad v_t^2 s'(v_t) < 1; \quad \xi_t \geq 0,$$

where

$$\mu_t \equiv \psi_t c_t s'(v_t) + \lambda U_c(c_t, h_t)[F'(h_t)h_t - F(h_t)] \frac{2s'(v_t) + v_t s''(v_t)}{[1 + s(v_t) + v_t s'(v_t)]^2} + \xi_t[2v_t s'(v_t) + v_t^2 s''(v_t)].$$

As before, ψ_t denotes the Lagrange multiplier associated with the feasibility constraint (14), $\lambda > 0$ denotes the Lagrange multiplier associated with the implementability constraint (16), and ξ_t denotes the Lagrange multiplier associated with the constraint $v_t^2 s'(v_t) < 1$. The satiation level of velocity, $\underline{v}$, does not represent a solution of this optimality condition. The reason is that at $v_t = \underline{v}$ the variable μ_t is negative, violating the optimality condition $\mu_t > 0$. To see this, note that μ_t is the sum of three terms. The first term, μ_t, $\psi_t c_t s'(v_t)$, is zero at $v_t = \underline{v}$ because $s'(\underline{v}) = 0$. Similarly, the third term, μ_t, μ_t, $\xi_t[2v_t s'(v_t) + v_t^2 s''(v_t)]$, is zero because ξ_t is zero, as the constraint $1 - v_t^2 s'(v_t)$ does not bind at $\underline{v}$. Finally, the second term of μ_t, $\lambda U_c(c_t, h_t)[F'(h_t)h_t - F(h_t)] \frac{2s'(v_t)+v_t s''(v_t)}{[1+s(v_t)+v_t s'(v_t)]^2}$, is negative. This is because under decreasing returns to scale $F'(h_t)h_t - F(h_t)$ is negative, and because under the maintained assumptions regarding the form of the transactions technology $s''(v)$ is strictly positive at $\underline{v}$.[3] As a consequence, the Friedman rule fails to be Ramsey optimal, and the Ramsey equilibrium features a positive nominal interest rate and inflation exceeding β.

[3] It can be argued that the assumption $2s'(v) + vs''(v) > 0$ for all $v >$ $\underline{v}$is too restrictive, which implies that the nominal interest rate is a strictly increasing function of v for all $v > \underline{v}$ and, in particular, that the elasticity of the liquidity preference function at a zero nominal interest rate is finite. Suppose instead that the assumption in question is relaxed by assuming that it must hold only for $v > \underline{v}$ but not at $v = \underline{v}$. In this case, a potential solution to the first-order condition of the Ramsey problem with respect to v_t is $v = \underline{v}$ provided $s''(\underline{v}) = 0$.

The factor $F(h) - F'(h)h$, which is in part responsible for the failure of the Friedman rule, represents pure profits accruing to the owners of firms. These profits are not taxed under the assumed labor income tax regime. We interpret the finding of a positive opportunity cost of holding money under the Ramsey optimal policy as an indirect way for the government to tax profits. It can be shown that if the government was able to tax profits either at the same rate as labor income or at 100% — which is indeed the Ramsey optimal rate — then the Friedman rule would re-emerge as the optimal monetary policy (see Schmitt-Grohé & Uribe, 2004b). Similarly, the Friedman rule is optimal if one assumes that, in addition to labor income taxes, the government has access to consumption taxes (see Correia, Nicolini, & Teles, 2008).

As an illustration of the inflation bias introduced by the assumption of decreasing returns to scale, we numerically solve for the Ramsey allocation in a parameterized, calibrated version of the model. We adopt the numerical solution method developed in Schmitt-Grohé and Uribe (2004b), which delivers an exact numerical solution to the Ramsey problem. We adopt the following forms for the period utility function, the production function, and the transactions cost technology:

$$U(c,h) = \ln(c) + \theta \ln(1-h); \quad \theta > 0, \tag{18}$$

$$F(h) = h^{\alpha}; \quad \alpha \in (0,1], \tag{19}$$

and

$$s(v) = Av + B/v - 2\sqrt{AB}. \tag{20}$$

The assumed transactions cost function implies that the satiation level of velocity is $\underline{v} = \sqrt{B/A}$ and a demand for money of the form

$$\frac{M_t}{P_t} = \frac{c_{t.}}{\sqrt{\frac{B}{A} + \frac{1}{A}\frac{R_t - 1}{R_t}}}.$$

We set $\beta = 1/1.04$, $\theta = 2.90$, $A = 0.0111$, $B = 0.07524$, $g_t = 0.04$ for all t, which implies a share of government spending of about 20% prior to the adoption of the Ramsey policy, and $(M_{-1} + R_{-1}\,B_{-1})/P_0 = 0.13$, which amounts to about 62% of GDP prior to the adoption of the Ramsey policy. For more details of the calibration strategy, see Schmitt-Grohé and Uribe (2004b).

Table 2 displays the Ramsey optimal levels of inflation and the labor-income tax rate for a range of values of α between 0.7 and 1. When α equals unity, the production function exhibits constant returns to scale and the entire output is taxed at the rate τ^h. This is the case studied most often in the literature. Table 2 shows that in this case, the Friedman rule is optimal and implies deflation at 3.85%. As the curvature of the production function increases, the untaxed fraction of GDP, given by $1 - \alpha$, also increases, inducing the Ramsey planner to use inflation as an indirect tax on this portion of output. The table shows

Table 2 Decreasing Return to Scale, Imperfect Competition, Tax Evasion, and Deviations from the Friedman Rule

Decreasing returns labor share			**Monopolistic competition markup**			**Tax evasion underground**		
α	π	τ^h	$\frac{\eta}{1+\eta}$	π	τ^h	Share, $\frac{\bar{u}}{y}$	π	τ^h
1.00	−3.85	17.99	1.00	−3.85	17.99	0.00	−3.85	17.99
0.99	−3.82	18.08	1.05	−3.65	19.74	0.06	−3.65	19.21
0.95	−3.70	18.42	1.10	−3.32	21.55	0.12	−3.37	20.62
0.90	−3.53	18.87	1.15	−2.83	23.42	0.18	−2.94	22.28
0.85	−3.33	19.34	1.20	−2.12	25.36	0.24	−2.20	24.29
0.80	−3.11	19.84	1.25	−1.11	27.35	0.31	−0.71	26.74
0.75	−2.86	20.36	1.30	−0.40	29.38	0.38	−3.31	29.60
0.70	−2.58	20.91	1.35	−2.71	31.41	0.46	20.02	31.38

Note: π and R denote, respectively, the net rates of inflation and interest rate expressed in percent per annum.

that as the untaxed fraction of output increases from 0 ($\alpha = 1$) to 30% ($\alpha = 0.7$), the Ramsey-optimal rate of inflation rises from −3.85% to −2.6%.

If one believes that at most 10% of the GDP of developed economies goes untaxed, then the value of α that is reasonable for the question analyzed here would be about 0.9. This value of α implies an inflation bias of about 30 basis points. We interpret this finding as suggesting that the inflation bias introduced by the presence of untaxed output in the decreasing-returns model provides a poor explanation for the actual inflation targets, of 2% or higher, adopted by central banks around the world.

4.2 Imperfect competition

Even if the production technologies available to firms exhibit constant returns to scale, pure profits may result in equilibrium if product markets are imperfectly competitive. If, in addition, the government is unable to fully tax pure monopoly profits or unable to tax them at the same rate as it taxes labor income, then deviating from the Friedman rule may be desirable. This case is analyzed in Schmitt-Grohé and Uribe (2004b).

To introduce imperfect competition, we modify the model studied in Section 4.1 by assuming that consumption is a composite good made from a continuum of differentiated intermediate goods via a Dixit-Stiglitz aggregator. Each intermediate good is produced by a monopolistically competitive firm that operates a linear technology, $F(h) = h$, and that faces a demand function with constant price elasticity rj $\eta < -1$. It can be shown that the only equilibrium condition that changes vis-à-vis the model developed earlier in this section is the labor demand function (8), that now becomes

$$F'(h_t) = \frac{\eta}{1+\eta} w_t, \tag{21}$$

where $\eta/(1 + \eta) > 1$ denotes the gross markup of prices over marginal cost.

A competitive equilibrium in the imperfect-competition economy is a set of sequences $\{v_t, c_t, h_t, M_t, B_t, P_t\}_{t=0}^{\infty}$ satisfying Eq. (2); Eq. (4) holding with equality; and Eqs.(5), (7), (11), (13)–(15) and (21), given policies $\{R_t, \tau_t^h\}_{t=0}^{\infty}$, the exogenous process $\{g\}_{t=0}^{\infty}$, and the initial condition $M_{-1} + R_{-1}\ B_{-1}$.

The primal form of the competitive equilibrium is identical to the one given in Section 3.1, with the implementability constraint (16) replaced by:[4]

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\left\{U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t + \frac{U_c(c_t,h_t)}{1+s(v_t)+v_t s'(v_t)}\frac{h_t}{\eta}\right\}\\ = \frac{U_c(c_0,h_0)}{1+s(v_0)+v_0 s'(v_0)}\frac{R_{-1}B_{-1}+M_{-1}}{P_0}.\end{aligned} \tag{22}$$

This implementability constraint is closely related to the one that results in the case of decreasing returns to scale. In effect, the factor $h_t/(-\eta)$, which appears in the preceding expression, represents pure profits accruing to the monopolists in the present economy. In the economy with decreasing returns, profits also appear in the implementability constraint in the form $F(h_t) - F'(h_t)h_t$. It should therefore come as no surprise that under imperfect competition the Ramsey planner has an incentive to inflate above the level called for by the Friedman rule as a way to levy an indirect tax on pure profits. To see this more formally, we present the first-order condition of the Ramsey problem with respect to money velocity for any $t > 0$, which is given by

$$\mu_t(v_t - \underline{v}) = 0; \quad v_t \geq \underline{v}; \quad \mu_t \geq 0; \quad \xi_t(1 - v_t^2 s'(v_t)) = 0; \quad v_t^2 s'(v_t) < 1; \quad \xi_t \geq 0,$$

where

$$\mu_t \equiv \psi_t c_t s'(v_t) + \lambda\frac{U_c(c_t,h_t)}{\eta}\frac{2s'(v_t)+v_t s''(v_t)}{[1+s(v_t)+v_t s'(v_t)]^2} + \xi_t[2v_t s'(v_t) + v_t^2 s''(v_t)].$$

Noting that $\eta < 0$, it follows by the same arguments presented in the case of decreasing returns to scale that the satiation level of velocity, $\underline{v}$, does not represent a solution to this first-order condition. The Friedman rule fails to be Ramsey optimal and the optimal rate of inflation exceeds β.

The middle panel of Table 2 presents the Ramsey optimal policy choices for inflation and the labor tax rate in the imperfectly competitive model for different values of the gross markup of prices over marginal cost, $\eta/(1 + \eta)$. All other structural

[4] The proof of this statement is similar to the one presented in Section 1 of the Appendix. For a detailed derivation see Schmitt-Grohé and Uribe (2004b).

parameters take the same value as before. The case of perfect competition corresponds to a markup of unity. In this case, the Friedman rule is optimal and the associated inflation rate is -3.85%. For positive values of the markup, the optimal interest rate increases as does the optimal level of inflation. Empirical studies (e.g., Basu & Fernald, 1997)) indicate that in post-war U.S. data value-added markups are at most 25%, which, according to Table 2, would be associated with an optimal inflation rate of only -1.11%. This inflation rate is far below the inflation targets of 2% or higher maintained by central banks. To obtain an optimal rate of inflation that is in line with observed central bank targets, our calibrated model would require a markup exceeding 30%, which is on the high end of empirical estimates.

The reason a high level of markup induces a high optimal rate of inflation in this model is because a high markup generates large profits that the Ramsey planner taxes indirectly with the inflation tax. For instance, a markup of 35% is associated with a profit share of 25% of GDP. Again this number seems unrealistically high. Any mechanism that would either reduce the size of the profit share (fixed costs of production) or reduce the amount of profits distributed to households (profit taxes) would result in lower optimal rates of inflation. For instance, if profits were taxed at a 100% rate, or if the profit tax rate were set equal to the labor income tax rate, τ_t^h, (i.e., if the tax system consisted in a proportional income tax rate), the Friedman rule would reemerge as Ramsey optimal. (See Schmitt-Grohé & Uribe, 2004b.)

4.3 Tax evasion

Our third example of how the Friedman rule breaks in the presence of an incomplete tax system is perhaps the most direct illustration of this principle. In this example, there is an underground economy in which firms evade income taxes. The failure of the Friedman rule due to tax evasion is studied in Nicolini (1998) in the context of a cash-in-advance model with consumption taxes. To maintain continuity with our previous analysis, here we embed an underground sector in our transaction cost model with income taxation. Specifically, we modify the model of Section 3 by assuming that firms can hide an amount u_t of output from the tax authority, which implies that the income tax rate applies only to the amount $F(h_t) - u_t$. Thus, the variable u_t is a measure of the size of the underground economy. The maximization problem of the firm is then given by

$$F(h_t) - w_t h_t - \tau_t[F(h_t) - u_t].$$

We allow the size of the underground economy to vary with the level of aggregate activity by assuming that u_t is the following function of h_t

$$u_t = u(h_t).$$

The first-order condition associated with the firm's profit maximization problem is

$$F'(h_t) = w_t + \tau_t[F'(h_t) - u'(h_t)]$$

This expression shows that the presence of the underground economy makes the labor input marginally cheaper in the amount $\tau_t u'(h_t)$.

All other aspects of the economy are assumed to be identical to those of the economy of Section 3 without income taxation at the level of the household. We restrict attention to the case of a linearly homogeneous production technology of the form $F(h) = h$. It follows that when the size of the underground economy is zero ($u_t = 0$ for all t), the economy collapses to that of Section 3 and the optimal inflation rate is the one associated with the Friedman rule.

When the size of the underground economy is not zero, one can show that the Ramsey problem consists in maximizing the lifetime utility function (1) subject to the feasibility constraint

$$[1 + s(v_t)]c_t + g_t = h_t,$$

the implementability constraint

$$\sum_{t=0}^{\infty}\beta^t\left\{U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t - \frac{u(h_t) - u'(h_t)h_t}{1 - v'(h_t)}\left[\frac{U_c(c_t,h_t)}{1 + s(v_t) + v_t s'(v_t)} + U_h(c_t,h_t)\right]\right\}$$
$$= \frac{U_c(c_0,h_0)}{1 + s(v_0) + v_0 s'(v_0)}\frac{R_{-1}B_{-1} + M_{-1}}{P_0}$$

and the following familiar restrictions on money velocity

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

given $(R_{-1}B_{-1} + M_{-1})$ and P_0.

Letting $\psi_t > 0$ denote the Lagrange multiplier on the feasibility constraint, $\lambda > 0$ the Lagrange multiplier on the implementability constraint, and μ_t the Lagrange multiplier on the constraint $v_t > \underline{v}$, the first-order condition of the Ramsey problem with respect to v_t is given by

$$\mu_t = \psi_t s'(v_t)c_t - \lambda\frac{u(h_t) - u'(h_t)h_t}{1 - u'(h_t)}\frac{U_c(c_t,h_t)}{[1 + s(v_t) + v_t s'(v_t)]^2}[2s'(v_t) + v_t s''(v_t)], \tag{23}$$

where μ_t satisfies

$$\mu_t \geq 0, \quad \text{and} \quad \mu_t(v_t - \underline{v}) = 0. \tag{24}$$

In deriving these conditions, we do not include in the Lagrangean the constraint $v_t s'(v_t) < 1$, so one must verify its satisfaction separately.

Consider two polar cases regarding the form of the function u, linking the level of aggregate activity and the size of the underground economy. One case assumes that u is homogeneous of degree one. In this case, we have that $u(h) - u'(h)h = 0$ and the above optimality conditions collapse to

$$\psi_t s'(v_t)c_t(v_t - \underline{v}) = 0, \quad v_t \geq \underline{v}, \quad \psi_t c_t s'(v_t) \geq 0.$$

This expression is identical to (17). We have established that, given our assumption regarding the form of the transaction cost technology s, optimality condition (17) can only be satisfied if $v_t = \underline{v}$. That is, the only solution to the Ramsey problem is the Friedman rule. The intuition for this result is that when the underground economy is proportional to the above-ground economy, a proportional tax on the above-ground output is also a proportional tax on total output. Thus, from a fiscal point of view, it is as if there was no untaxed income.

The second polar case assumes that the size of the underground economy is independent of the level of aggregate activity; that is, $u(h_t) = \bar{u}$, where $\bar{u} > 0$ is a parameter. In this case, when v_t equals $\underline{v}$, optimality condition (23) implies that $\mu_t = -\lambda\, \bar{u}U_c(c_t, h_t)\underline{v}s''(\underline{v}) < 0$, violating optimality condition (24). It follows that the Friedman rule ceases to be Ramsey optimal. The intuition behind this result is that in this case firms operating in the underground economy enjoy a pure rent given by the amount of taxes that they manage to evade. The base of the evaded taxes is perfectly inelastic with respect to both the tax rate and inflation, and given by $\bar{u}$. The government attempts to indirectly tax these pure rents by imposing an inflation tax on consumption.

The failure of the Friedman rule in the presence of an underground sector holds more generally. For instance, the result obtains when the function u is homogeneous of any degree ϕ less than unity. To see this, note that in this case when $v_t = \underline{v}$, Eq. (23) becomes $\mu_t = -\lambda \frac{u(h_t)(1-\phi)}{1-\phi u(h_t)/h_t} U_c(c_t, h_t)\, \underline{v}\, s''(\underline{v}) < 0$. In turn, the negativity of μ_t contradicts optimality condition (24). Consequently, v_t must be larger than $\underline{v}$ and the Friedman rule fails to hold.

The right panel of Table 2 presents the Ramsey optimal inflation rate and labor income tax rate as a function of share of the underground sector in total output. In these calculations we assume that the size of the underground economy is insensitive to changes in output ($u'(h) = 0$). All other functional forms and parameter values are as assumed in Section 4.1. Nicolini (1998) reported estimates for the size of the underground economy in the U.S. of at most 10%. Table 2 shows that for a share of underground economy of this magnitude the optimal rate of inflation is only 50 basis points above the one associated with the Friedman rule. This implies that in the context of this model tax evasion provides little incentive for the monetary authority to inflate.

From the analysis of these three examples we conclude that it is difficult, if not impossible, to explain observed inflation targets as the outcome of an optimal monetary and fiscal policy problem through the lens of a model in which the incentives to inflate stem from the desire to mend an ill-conceived tax system.

In the next section we present an example in which the Ramsey planner has an incentive to inflate that is purely monetary in nature and unrelated to fiscal policy considerations.

5. A FOREIGN DEMAND FOR DOMESTIC CURRENCY AND THE OPTIMAL RATE OF INFLATION

More than half of U.S. currency circulates abroad. Porter and Judson (1996) estimated that at the end of 1995 \$200–250 billion of the \$375 billion of U.S. currency in circulation outside of banks was held abroad. The foreign demand for U.S. currency has remained strong across time. The 2006 Treasury, Federal Reserve, and Secret Service report on the use of U.S. currency abroad, issued a decade after the publication of Porter and Judson (1996), estimated that as of December 2005 about \$450 billion of the \$760 billion of circulated U.S. banknotes are held in other countries.

The estimated size of the foreign demand for U.S. currency suggests that much of the seignorage income of the United States is generated outside of its borders. Therefore, a natural question is whether a country's optimal rate of inflation is influenced by the presence of a foreign demand for its currency. In this section we address this issue within the context of a dynamic Ramsey problem. We show that the mere existence of a foreign demand for domestic money can, under plausible parameterizations, justify sizable deviations from the rate of inflation associated with the Friedman rule. The basic intuition behind this finding is that adherence to the negative rate of inflation associated with the Friedman rule would represent a welfare-decreasing transfer of real resources by the domestic economy to the rest of the world, as nominal money balances held abroad increase in real terms at the rate of deflation. A benevolent government weighs this cost against the benefit of keeping the opportunity cost of holding money low to reduce transactions costs for domestic agents. Our analytical results show that this trade-off is resolved in favor of deviating from the Friedman rule. Indeed, our quantitative analysis suggests that for plausible calibrations the optimal rate of inflation is positive. The question of how a foreign demand for money affects the optimal rate of inflation is studied in Schmitt-Grohé and Uribe (2009a_. We follow this paper closely in this section.

5.1 The model

We consider a variation of the constant-returns-to-scale, perfectly-competitive, monetary economy of Section 3 augmented with a foreign demand for domestic currency. Specifically, assume that the foreign demand for real domestic currency, M_t^f/P_t, is a function of the level of foreign aggregate activity, denoted y_t^f, and the domestic nominal interest rate. Formally, the foreign demand for domestic currency is implicitly given by

$$\left(v_t^f\right)^2 \tilde{s}'\left(v_t^f\right) = \frac{R_t - 1}{R_t}, \tag{25}$$

where v_t^f is defined as

$$v_t^f = \frac{P_t y_t^f}{M_t^f}. \tag{26}$$

The transactions cost technology $\tilde{s}$ is assumed to satisfy the same properties as the domestic transactions cost function s.

As in previous sections, we assume that the government prints money; issues nominal, one-period bonds; and levies taxes to finance an exogenous stream of public consumption, denoted g_t, and interest obligations on the outstanding public debt. Accordingly, the government's sequential budget constraint is given by

$$M_t + M_t^f + B_t = M_{t-1} + M_{t-1}^f + R_{t-1}B_{t-1} + P_t g_t - P_t \tau_t^h w_t h_t, \tag{27}$$

where M_t now denotes the stock of money held domestically. Combining this expression with the household's sequential budget constraint, given by $P_t c_t[1 + s(v_t)] + M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_t\left(1 - \tau_t^h\right)w_t h_t$ yields the following aggregate resource constraint

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{M_t^f - M_{t-1}^f}{P_t}, \tag{28}$$

where we are using the fact that with perfect competition in product markets and a constant returns to scale production function $w_t h_t = F(h_t)$. It is clear from this resource constraint that the domestic economy collects seignorage revenue from foreigners whenever nominal money balances held by foreigners increase; that is, whenever $M_t^f > M_{t-1}^f$. This would happen in an inflationary environment characterized by a constant foreign demand for domestic real balances. Conversely, the domestic economy transfers real resources to the rest of the world whenever the foreign demand for domestic currency shrinks $M_t^f < M_{t-1}^f$, as would be the case in a deflationary economy facing a constant foreign demand for domestic real balances.

A competitive equilibrium is a set of sequences $\left\{v_t, w_t, v_t^f, c_t, h_t, M_t, M_t^f, B_t, P_t\right\}_{t=0}^{\infty}$ satisfying Eq. (2); Eq. (4) holding with equality; Eqs. (5), (7), (8), (11), (13), and (25)–(28), given policies $\left\{R_t, \tau_t^h\right\}_{t=0}^{\infty}$, the exogenous sequences $\left\{g_t, y_t^f\right\}_{t=0}^{\infty}$, and the initial conditions $M_{-1} + R_{-1}\, B_{-1} > 0$ and M_{-1}^f.

To characterize the optimal rate of inflation it is convenient to first derive the primal form of the competitive equilibrium. Given the initial conditions $(R_{-1}B_{-1} + M_{-1})$ and M^f_{-1} and the initial price level P_0, sequences $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ satisfy the feasibility condition

$$[1 + s(v_0)]c_0 + g_0 = F(h_0) + \frac{\gamma_0^f}{\chi(v_0)} - \frac{M^f_{-1}}{P_0} \tag{29}$$

in period 0 and

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{\gamma_t^f}{\chi(v_t)} - \frac{\gamma_{t-1}^f}{\chi(v_{t-1})}\left(1 - v_{t-1}^2 s'(v_{t-1})\right)\frac{U_c(c_{t-1}, h_{t-1})}{\gamma(v_{t-1})}\frac{\gamma(v_t)}{\beta U_c(c_t, h_t)}, \tag{30}$$

for all $t > 0$, the implementability constraint

$$\sum_{t=0}^{\infty}\beta^t\{U_c(c_t, h_t)c_t + U_h(c_t, h_t)h_t\} = \frac{U_c(c_0, h_0)}{1 + s(v_0) + v_0 s'(v_0)}\frac{R_{-1}B_{-1} + M_{-1}}{P_0}, \tag{31}$$

and

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

if and only if they also satisfy the set of equilibrium conditions (2); Eq. (4) holding with equality; Eqs. (5), (7), (8), (11), (13), and (25)–(28), where the function

$$v_t^f = \chi(v_t) \tag{32}$$

is implicitly defined by $v^2 s'(v) - (v^f)^2 \tilde{s}'(v^f) = 0$. Section 2 of the Appendix presents the proof of this statement of the primal form of the competitive equilibrium.

5.2 Failure of the Friedman rule

The government is assumed to be benevolent toward domestic residents. This means that the welfare function of the government coincides with the lifetime utility of the domestic representative agent, and that it is independent of the level of utility of foreign residents. The Ramsey problem then consists in choosing a set of strictly positive sequences $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ to maximize the utility function (1) subject to Eqs. (29)–(31), $v_t \geq v$, and $v_t^2 s'(v_t) < 1$, given $R_{-1}\, B_{-1} + M_{-1}$, M^f_{-1}, and P_0.

To simplify notation express the feasibility constraint (30) as $H(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) = 0$ and the implementability constraint (31) as $\sum_{t=0}^{\infty}\beta^t K(c_t, h_t) = A(c_0, h_0, v_0)$. Let the Lagrange multiplier on the feasibility constraint (30) be denoted by ψ_t, the Lagrange multiplier on the implementability constraint (31) be denoted by λ, and the Lagrange multiplier on the constraint $v_t \geq \underline{v}$ be denoted by μ_t. Then, for any $t > 0$, the first-order conditions of the Ramsey problem are

$$\begin{aligned} &U_c(c_t, h_t) + \lambda \mathrm{K}_c(c_t, h_t) + \psi_t H_1(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) \\ &+ \beta \psi_{t+1} H_2(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) = 0 \end{aligned} \tag{33}$$

$$\begin{aligned} &U_h(c_t, h_t) + \lambda \mathrm{K}_h(c_t, h_t) + \psi_t H_3(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) \\ &+ \beta \psi_{t+1} H_4(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) = 0 \end{aligned} \tag{34}$$

$$\psi_t H_5(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) + \beta \psi_{t+1} H_6(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) + \mu_t = 0, \tag{35}$$

$$(v_t - \underline{v})\mu_t = 0; \quad \mu_t \geq 0; \quad v_t \geq \underline{v}\,. \tag{36}$$

We do not include the constraint $v_t^2 s'(v_t) < 1$ in the Lagrangean. Therefore, we must check that the solution to the above system satisfies this constraint.

Because this economy collapses to the one studied in Section 3 when the foreign demand for domestic currency is zero; that is, when $\gamma_t^f = 0$, it follows immediately that in this case the Friedman rule is Ramsey optimal. We first establish analytically that the Friedman rule ceases to be Ramsey optimal in the presence of a foreign demand for domestic currency, that is, when $\gamma_t^f > 0$. To facilitate the exposition, as in previous sections, we restrict attention to the steady state of the Ramsey equilibrium. In other words, we restrict attention to solutions to Eqs. (30) and (33)–(36) in which the endogenous variables c_t, h_t, v_t, ψ_t and μ_t are constant given constant levels for the exogenous variables g_t and γ_t^f. Further, absent an estimate of the foreign demand for domestic currency, throughout this section, we assume that $\chi(v) = v$, which implies identical relationships between the nominal interest rate and domestic-money velocity in the domestic and the foreign economies. To establish the failure of the Friedman rule when $\gamma_t^f > 0$, we show that a Ramsey equilibrium in which v_t equals $\underline{v}$ is impossible. In the steady state, the optimality condition (35) when evaluated at $v_t = \underline{v}$ becomes:

$$\psi \frac{\gamma^f}{\chi(\underline{v})} s''(\underline{v})\, \underline{v} \left(1 - \frac{1}{\beta} + \underline{v}\right) + \mu = 0.$$

For the reasons given in Section 3, the Lagrange multiplier ψ is positive. Under our maintained assumptions regarding the transactions cost technology, $s''(\underline{v})$ is also positive.[5] Under reasonable calibrations, the constant $1/\beta - 1$, which equals the steady-state real interest rate, is smaller than the velocity level $\underline{v}$. Then, the first term in the previous sum is positive. This implies that the multiplier μ must be negative, which violates optimality condition (36). We conclude that in the presence of a foreign demand for domestic currency, if a Ramsey equilibrium exists, it involves a deviation from the Friedman rule.

The intuition behind this result is that the presence of a foreign demand for domestic currency introduces an incentive for the fiscal authority to inflate in order to extract

[5] See the discussion in footnote 3.

resources, in the form of seignorage, from the rest of the world (whose welfare does not enter the domestic planner's objective function). Indeed, at any negative inflation rate (and, most so at the level of inflation consistent with the Friedman rule), the domestic country actually derives negative seignorage income from the rest of the world, because foreign money holdings increase in real value as the price level falls. On the other hand, levying an inflation tax on foreign money holdings comes at the cost of taxing domestic money holdings as well. In turn, the domestic inflation tax entails a welfare loss, because domestic households must pay elevated transaction costs as they are forced to economize on real balances. Thus, the Ramsey planner faces a trade-off between taxing foreign money holdings and imposing transaction costs on domestic residents. We have demonstrated analytically that the resolution of this trade-off leads to an inflation rate above the one called for by Friedman's rule. We now turn to the question of how large the optimal deviation from the Friedman rule is under a plausible calibration of our model.

5.3 Quantifying the optimal deviation from the Friedman rule

To gauge the quantitative implications of a foreign demand for money for the optimal rate of inflation, we parameterize the model and solve numerically for the steady state of the Ramsey equilibrium. We adopt the functional form given in equation (18) for the period utility function and the functional form given in Eq. (20) for the transactions cost technology. As in Section 3, we set $\beta = 1/1.04$, $\theta = 2.90$, $B = 0.07524$, and $g_t = 0.04$ for all t. We set $\gamma^f = 0.06$ and $A = 0.0056$ to match the empirical regularities that about 50% of U.S. currency (or about 26 of M1) is held outside of the United States and that the M1-to-consumption ratio is about 29%. Finally, to make the Ramsey steady state in the absence of a foreign demand for money approximately equal to the one of the economy considered in Section 3, we set the level of debt in the Ramsey steady state to 20% of GDP. This debt level implies that the pre-Ramsey reform debt-to-output ratio in the economy without a foreign demand for domestic currency and with a pre-reform inflation rate of 4.2% is about 44%. The reason the Ramsey steady-state level of debt is much lower than the pre-Ramsey-reform level is because the reform induces a drop in expected inflation of about 8%, which causes a large asset substitution away from government bonds and toward real money balances. The overall level of government liabilities (money plus bonds) is relatively unaffected by the Ramsey reform.

We develop a numerical algorithm that delivers the exact solution to the steady state of the Ramsey equilibrium. The mechanics of the algorithm are

1. Pick a positive value of λ.
2. Given this value of λ solve the nonlinear system (30) and (33)–(36) for c, h, v, ψ, and μ.
3. Calculate w from Eq. (8), τ^h from Eq. (13), R from Eq. (5), π from Eq. (7), v^f from Eq. (32), M_t/P_t from Eq. (2), and M_t^f/P_t from Eq. (26).

Table 3 Ramsey policy with foreign demand for domestic currency

	$\frac{M^f}{M^f+M}$	$\frac{M^f+M}{P_c}$	π	R	τ^h
No foreign demand: $\gamma^f = 0$	0.00	0.27	−3.85	0.00	17.56
Baseline calibration: $\gamma^f = 0.06$	0.22	0.26	2.10	6.18	16.15
Higher foreign demand: $\gamma^f = 0.1$	0.32	0.24	10.52	14.94	14.64
Low domestic demand: $A = 0.0014$	0.22	0.13	2.11	6.19	16.33
High interest elasticity: $B = 0.0376$	0.22	0.37	−0.96	3.00	16.95
High debt-to-output ratio: $\frac{B}{P_y} = 0.50$	0.22	0.26	2.21	6.30	17.50
Lump-sum taxes	0.20	0.27	0.85	4.88	0.00
Lump-sum taxes and $g_t = 0$	0.19	0.27	0.59	4.62	—

Note: The baseline calibration is: A = 0.0056, B = 0.07524, $\frac{B}{P_y} = 0.2$, $\gamma^f = 0.06$. The interest rate, R, and the inflation rate, π, are expressed in percent per annum, and the income tax rate, τ^h, is expressed in percent.

4. Calculate the steady-state debt-to-output ratio, which we denote by $s_d \equiv B_t/(P_t y_t)$, from Eq. (27), taking into account that $y = h$.
5. If s_d is larger than the calibrated value of 0.2, lower λ. If, instead, s_d is smaller than the calibrated value of 0.2, then increase the value of λ.
6. Repeat steps 1–5 until s_d has converged to its calibrated value.

Table 3 presents our numerical results. The first line of the table shows that when foreign demand for domestic currency is zero, which we capture by setting $\gamma^f = 0$, then as we have shown analytically in Section 3, the Friedman rule is Ramsey optimal; that is, the nominal interest rate is zero in the steady state of the Ramsey equilibrium. The inflation rate is −3.85% and the income tax rate is about 18%. In this case, because the foreign demand for domestic currency is zero, the domestic government has no incentives to levy an inflation tax, as it would generate no revenues from the rest of the world but would hurt domestic residents by elevating the opportunity costs of holding money. The second row of the table considers the case that the foreign demand for domestic currency is positive. In particular, we set $\gamma^f = 0.06$ and obtain that in the Ramsey steady state the ratio of foreign currency to total money is 22% and that total money holdings are 26% of consumption. Both figures are broadly in line with observations in the U.S. economy. Table 3 shows, in line with the analytical results previously obtained, that the Ramsey optimal rate of interest is positive, that is, the Friedman rule is no longer optimal. Of greater interest, however, is the size of the deviation from the Friedman rule. Table 3 shows that the Ramsey optimal inflation rate is 2.10% per year, about 6 percentage points higher than the value obtained in the absence of a foreign demand for domestic currency. The optimal rate of interest now is 6.2%. When we increase foreign demand for domestic currency by assuming a larger value of foreign activity, $y^f = 0.1$, then the share of foreign holdings of domestic currency in total money increases by 10 percentage points to 0.32 and the Ramsey optimal inflation rate is more than 10% per year. In this calibration, the benefit from collecting an inflation tax from foreign holdings of currency appears to strongly dominate the costs that such a high inflation tax represents for domestic agents in terms of a more distorted consumption–leisure choice and elevated transaction costs. The larger inflation tax revenues relax the budget constraint of the government allowing for a decline in the Ramsey optimal tax rate of about 1.5 percentage points.

Line 4 of Table 3 considers a calibration that implies a weaker demand for money both domestically and abroad. Specifically, we lower the coefficient A in the transactions cost function by a factor of 4. Because the demand for money is proportional to the square root of A, this parameter change implies that the ratio of money to consumption falls by a factor of two. In the Ramsey steady state, the money-to-consumption ratio falls from 26 to 13%. The relative importance of foreign demand for money is unchanged. It continues to account for 22% of total money demand. The optimal rate of inflation is virtually the same as in the baseline case. The reason the inflation tax is virtually unchanged in this case is because the reduction in A induces proportional declines in both the domestic and the foreign demands for domestic currency. The decline in foreign money demand is equivalent to a decline in y^f, therefore inducing the Ramsey planner to lower the rate of inflation. At the same time, the decline in the domestic demand for money reduces the cost of inflation for domestic agents, inducing the Ramsey planner to inflate more. In our parameterization, these two opposing effects happen to offset each other almost exactly.

Line 5 of Table 3 analyzes the sensitivity of our results to raising the interest elasticity of money demand, which we capture by reducing the parameter B of the transaction cost function to half its baseline value. Under a higher interest elasticity the Ramsey optimal rate of interest and inflation are lower than in the baseline case. The nominal interest rate falls from 6 to 3% and the inflation rate falls from about 2% to −1%. In this case while the Ramsey policy deviates from the Friedman rule, the deviation is not large enough to render positive inflation Ramsey optimal. The last line of Table 3 shows that our results change very little when we increase the steady-state debt level. We conclude from the results presented in Table 3 that the trade-off between collecting seignorage from foreign holders of domestic currency and keeping the opportunity cost of holding money low for domestic agents is resolved in favor of collecting seignorage income from foreign holdings of domestic currency.

5.4 Lump-sum taxation

The reason the benevolent government finds it desirable to deviate from the Friedman rule in the presence of a foreign demand for currency is to not entirely finance its budget with seignorage revenue extracted from foreign residents. Rather, the government imposes an inflation tax on foreign residents to increase the total amount of resources available to domestic residents for consumption. To show that this is indeed the correct interpretation of our results, we now consider a variation of the model in which the government can levy lump-sum taxes on domestic residents. Specifically, we assume that the labor income tax rate τ_t^h is zero at all times, and that the government sets lump-sum taxes to ensure fiscal solvency. A competitive equilibrium in the economy with lump-sum taxes is then given by sequences $\left\{v_t,\ v_t^f,\ c_t,\ h_t,\ M_t,\ M_t^f,\ P_t,\ w_t\right\}_{t=0}^{\infty}$ satisfying Eqs. (2), (5), (6), (7), (8), (11), (25), (26), and (28), given an interest rate sequence $\{R_t\}_{t=0}^{\infty}$, and the exogenous sequences $\left\{y_t^f,\ g_t\right\}_{t=0}^{\infty}$.

One can show that, given the initial condition M_{-1}^f and the initial price level P_0, sequences $\{c_t,\ h_t,\ v_t\}_{t=0}^{\infty}$ satisfy the feasibility conditions (29) and (30), the labor supply equation

$$-\frac{U_h(c_t,\ h_t)}{U_c(c_t,\ h_t)}=\frac{1}{1+s(v_t)+v_t s'(v_t)} \tag{37}$$

and

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

if and only if they also satisfy the set of equilibrium conditions (2), (5), (6), (7), (8), (11), (25), (26), and (28). This primal form of the equilibrium conditions is essentially the same as the one associated with the economy with distortionary taxes and government spending except that the implementability constraint is replaced by Eq. (37), which states that in equilibrium labor demand must equal labor supply. Noting that Eq. (37) appears in both the standard and the primal forms of the competitive equilibrium, it follows that the proof of the above statement is a simplified version of the one presented in Section 2 of the Appendix. The Ramsey problem then consists in maximizing the utility function (1) subject to the feasibility constraints (29) and (30), the labor market condition (37), and the restrictions $v_t \geq \underline{v}$ and $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$, given P_0 and M_{-1}^f.

Line 7 of Table 3 presents the steady state of the Ramsey equilibrium in the economy with lump-sum taxes. All parameters of the model are calibrated as in the economy with distortionary taxes. The table shows that the optimal rate of inflation equals 0.85% per year. This means that the presence of a foreign demand for money gives rise to an optimal inflation bias of about 5 percentage points above the level of inflation called for by the Friedman rule. This inflation bias emerges even though the government can resort to lump-sum taxes to finance its budget. The optimal inflation bias is smaller than in the case with distortionary taxes. This is because distortionary taxes, through their depressing effect on employment and output, make the pre-foreign-seignorage level of consumption lower, raising the marginal utility of wealth, and as a result provide bigger incentives for the extraction of real resources from the rest of the world.

The last row of Table 3 displays the steady state of the Ramsey equilibrium in the case in which government consumption equals zero at all times ($g_t = 0$ for all t). All other things equal, the domestic economy has access to a larger amount of resources than the economy with positive government consumption. As a result, the government has fewer incentives to collect seignorage income from the rest of the world. This is reflected in a smaller optimal rate of inflation of 0.59%. It is remarkable, however, that even in the absence of distortionary taxation and in the absence of public expenditures, the government finds it optimal to deviate from the Friedman rule. Notice that in the absence of a foreign demand for money, this economy is identical to the one analyzed in Section 2. It follows that in the absence of a foreign demand for money the Friedman rule would be Ramsey optimal and the optimal inflation rate would be negative 3.8%. The finding that optimal inflation is indeed positive when a foreign demand for money is added to this simple model clearly shows that fiscal considerations play no role in determining that the optimal rate of inflation is positive. The ultimate purpose of positive interest rates in the presence of a foreign demand for money is the extraction of real resources from the rest of the world for private domestic consumption.

The numerical results of this section suggest that an inflation target of about 2% per annum may be rationalized on the basis of an incentive to tax foreign holdings of domestic currency. This argument could, in principle, be raised to explain inflation targets observed in countries whose currencies circulate widely outside of their borders, such as the United States and the Euro Area. However, the fact that a number of developed countries whose currencies are not used outside of their geographic borders, such as Australia, Canada, and New Zealand, also maintain inflation targets of about 2% per year. This indicates that the reason inflation targets in the developed world are as high as observed may not originate from the desire to extract seignorage revenue from foreigners.

The family of models we have analyzed up to this point have two common characteristics: one is that a transactions demand for money represents the only source of monetary non-neutrality. The second characteristic is full flexibility of nominal prices. We have demonstrated, through a number of examples, that within the limits imposed by these two theoretical features it is difficult to rationalize why most central banks in the developed world have explicitly or implicitly set for themselves inflation targets significantly above zero. We therefore turn next to an alternative class of monetary models in which additional costs of inflation arise from the presence of sluggish price adjustment. As we will see in this class of model, quite different trade-offs than the ones introduced thus far shape the choice of the optimal rate of inflation.

6. STICKY PRICES AND THE OPTIMAL RATE OF INFLATION

At the heart of modern models of monetary non-neutrality is the New Keynesian Phillips curve, which defines a dynamic trade-off between inflation and marginal costs that arises in dynamic general equilibrium model economies populated by utility-maximizing households and profit-maximizing firms augmented with some kind of rigidity in the adjustment of nominal product prices. The foundations of the New Keynesian Phillips curve were laid by Calvo (1983) and Rotemberg (1982). Woodford (1996, 2003) and Yun (1996) completed the development of the New Keynesian Phillips curve by introducing optimizing behavior on the part of firms facing Calvo-type dynamic nominal rigidities.

The most important policy implication of models featuring a new Keynesian Phillips curve is the optimality of price stability. Goodfriend and King (1997) provided an early presentation of this result. This policy implication introduces a sharp departure from the flexible-price models discussed in previous sections, in which optimal monetary policy gravitates not toward price stability, but toward price deflation at the real rate of interest.

We start by analyzing a simple framework within which the price-stability result can be obtained analytically. To this end, we remove the money demand friction from the model of Section 2 and instead introduce costs of adjusting nominal product prices. In the resulting model, sticky prices represent the sole source of nominal friction. The model incorporates capital accumulation and uncertainty both to stress the generality of the price stability result and because these two features will be of use later in this chapter.

6.1 A sticky-price model with capital accumulation

Consider an economy populated by a large number of households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t,\ h_t), \tag{38}$$

where E_t denotes the expectations operator conditional on information available at time t. Other variables and symbols are as defined earlier. Households collect income from supplying labor and capital services to the market and from the ownership of firms. Labor income is given by $w_t h_t$, and income from renting capital services is given by $r_t^k k_t$, where r_t^k and k_t denote the rental rate of capital and the capital stock, respectively. Households have access to complete contingent claims markets. Specifically, in every period t households can purchase nominal state-contingent assets. The period t price of a stochastic payment D_{t+1} is given by $E_t r_{t,\ t+1} D_{t+1}$, where $r_{t,\ s}$ is a nominal stochastic discount factor such that the period t value of a state-contingent payment D_s occurring in period s is $E_t r_{t,\ s} D_s$. The household's period-by-period budget constraint takes the form

$$c_t + i_t + E_t r_{t,t+1} \frac{D_{t+1}}{P_t} = \frac{D_t}{P_t} + \left(1 + \tau_t^D\right)\left[w_t h_t + r_t^k k_t\right] + \phi_t - \tau_t^L \tag{39}$$

Here, i_t denotes gross investment, ϕ_t denotes profits received from the ownership of firms, τ_t^D denotes the income tax rate, and τ_t^L denotes lump-sum taxes. The capital stock is assumed to depreciate at the constant rate δ. The evolution of capital is given by

$$k_{t+1} = (1-\delta)k_t + i_t. \tag{40}$$

Households are also assumed to be subject to a borrowing limit of the form $\lim_{s\to\infty} E_t r_{t,s} D_s \geq 0$, which prevents them from engaging in Ponzi schemes.

The household's problem consists of maximizing the utility function (38) subject to Eqs. (39), (40), and the no-Ponzi-game borrowing limit. The first-order conditions associated with the household's problem are

$$\begin{aligned} -\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} &= \left(1 - \tau_t^D\right) w_t, \\ U_c(c_t, h_t) &= \beta E_t U_c(c_{t+1}, h_{t+1})\left[\left(1 - \tau_{t+1}^D\right) r_{t+1}^k + (1-\delta)\right] \\ U_c(c_t, h_t) r_{t,t+1} &= \beta \frac{U_c(c_{t+1}, h_{t+1})}{\pi_{t+1}}. \end{aligned} \tag{41}$$

Final goods, denoted $a_t \equiv c_t + i_t$, are assumed to be a composite of a continuum of differentiated intermediate goods, a_{it}, $i \in [0,1]$, produced via the aggregator function

$$a_t = \left[\int_0^1 a_{it}{}^{1-1/\eta} di\right]^{1/(1-1/\eta)},$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of intermediate goods. The demand for intermediate good a_{it} is then given

$$a_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} a_t,$$

where P_t is a nominal price index defined as

$$P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}. \tag{42}$$

Each good's variety $i \in [0,1]$ is produced by a single firm in a monopolistically competitive environment. Each firm i produces output using as factor inputs capital services, k_{it}, and labor services, h_{it}, both of which are supplied by households in a perfectly competitive fashion. The production technology is given by

$$z_t F(k_{it}, h_{it}) - \chi,$$

where the function F is assumed to be homogeneous of degree one, concave, and strictly increasing in both arguments. The variable Z_t denotes an exogenous, aggregate productivity shock. The parameter χ introduces fixed costs of production. Firms are assumed to satisfy demand at the posted price, that is,

$$z_t F(k_{it}, h_{it}) - \chi \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} a_t. \tag{43}$$

Profits of firm i at date t are given by

$$\frac{P_{it}}{P_t} a_{it} - r_t^k k_{it} - w_t h_{it}.$$

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, and k_{it} to maximize the present discounted value of profits, given by

$$E_t \sum_{s=t}^{\infty} r_{t,s} P_s \left[\frac{P_{is}}{P_s} a_{is} - r_s^k k_{is} - w_s h_{is},\right]$$

subject to constraint (43). Then, letting $r_{t,s}$ P_s mc_{is} be the Lagrange multiplier associated with constraint (43), the first-order conditions of the firm's maximization problem with respect to labor and capital services are, respectively,

$$\mathrm{mc}_{it} z_t F_h(k_{it}, h_{it}) = w_t$$

and

$$\mathrm{mc}_{it} z_t F_k(k_{it}, h_{it}) = r_t^k.$$

It is clear from these expressions that the Lagrange multiplier mc_{it} reflects the marginal cost of production of variety i in period t. Notice that because all firms face the same factor prices and because they all have access to the same production technology with F homogeneous of degree one, the capital-labor ratio, k_{it}/h_{it} and marginal cost, mc_{it}, are identical across firms. Therefore, we will drop the subscript i from mc_{it}.

Prices are assumed to be sticky à la Calvo (1983), Woodford (1996), and Yun (1996). Specifically, each period, a fraction $\alpha \in [0,1)$ of randomly picked firms, is not allowed to change the nominal price of the good it produces; that is, each period, a fraction α of firms, must charge the same price as in the previous period. The remaining $(1-\alpha)$ firms choose prices optimally. Suppose firm i gets to pick its price in period t, and let $\widetilde{P}_{it}$ denote the chosen price. This price is set to maximize the expected present discounted value of profits. That is, $\widetilde{P}_{it}$ maximizes $E_t\sum_{s=t}^{\infty} r_{t,s}P_s\alpha^{s-t}\left\{\left[\left(\frac{\widetilde{P}_{it}}{P_s}\right)^{1-\eta}a_s - r_s^k k_{is} - w_s h_{is}\right] + \mathrm{mc}_s\left[z_sF(k_{is},h_{is}) - \chi - \left(\frac{\widetilde{P}_{it}}{P_s}\right)^{-\eta}a_s\right]\right\}$.

The first-order condition associated with this maximization problem is

$$E_t\sum_{s=t}^{\infty} r_{t,s}\alpha^{s-t}\left(\frac{\widetilde{P}_{it}}{P_s}\right)^{-1-\eta}a_s\left[\mathrm{mc}_s - \frac{\eta-1}{\eta}\frac{\widetilde{P}_{it}}{P_s}\right] = 0.$$

According to this expression, firms whose price is free to adjust in the current period pick a price level such that a weighted average of current and future expected differences between marginal costs and marginal revenue equals zero. Moreover, it is clear from this optimality condition that the chosen price $\widetilde{P}_{it}$ is the same for all firms that can reoptimize their price in period t. We can therefore drop the subscript i from $\widetilde{P}_{it}$. We link the aggregate price level P_t to the price level chosen by the $(1-\alpha)$ firms that reoptimize their price in period t, $\widetilde{P}_t$. To this end, we write the definition of the aggregate price level given in Eq. (42) as follows

$$P_t^{1-\eta} = \alpha P_{t-1}^{1-\eta} + (1-\alpha)\widetilde{P}_t^{1-\eta}.$$

Letting $\widetilde{p}_t \equiv \frac{\widetilde{P}_t}{P_t}$ denote the relative price of goods produced by firms that reoptimize their price in period t and $\pi_t \equiv P_t/P_{t-1}$ denote the gross rate of inflation in period t, the previous expression can be written as

$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\widetilde{p}_t^{1-\eta}.$$

We derive an aggregate resource constraint for the economy by imposing market clearing at the level of intermediate goods. Specifically, the market clearing condition in the market for intermediate good i is given by

$$z_tF(k_{it},h_{it}) - \chi = a_{it}.$$

Taking into account that $a_{it} = a_t\left(\frac{P_{it}}{P_t}\right)^{-\eta}$, and the capital labor ratio k_{it}/h_{it} is independent of i, and that the function F is homogeneous of degree of one, we can integrate the preceding market clearing condition over all goods i to obtain

$$h_tz_tF\left(\frac{k_t}{h_t},1\right) - \chi = s_ta_t,$$

where $h_t \equiv \int_0^1 h_{it}di$ and $k_t \equiv \int_0^1 k_{it}di$ denote the aggregate levels of labor and capital services in period t and $s_t \equiv \int_0^1\left(\frac{P_{it}}{P_t}\right)^{-\eta}di$ is a measure of price dispersion. To complete the aggregation of the model we express the variable s_t recursively as follows

$$
\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \\
&= \int_{1-\alpha} \left(\frac{\widetilde{P}_t}{P_t}\right)^{-\eta} di \;+\; \int_{\alpha} \left(\frac{P_{it-1}}{P_t}\right)^{-\eta} di \\
&= (1-\alpha)\widetilde{p}_t^{\,-\eta} + \left(\tfrac{P_{t-1}}{P_t}\right)^{-\eta} \int_{\alpha} \left(\frac{P_{it-1}}{P_{t-1}}\right)^{-\eta} di \\
&= (1-\alpha)\widetilde{p}_t^{\,-\eta} + \alpha \pi_t^{\eta} s_{t-1}.
\end{aligned}
$$

The state variable s_t measures the resource costs induced by the inefficient price dispersion present in the Calvo-Woodford-Yun model in equilibrium. Two observations are in order about the dispersion measure s_t. First, s_t is bounded below by 1. Second, in an economy where the nonstochastic level of inflation is zero; that is, when $\pi = 1$, there is no price dispersion in the long-run. So $s = 1$ in the deterministic steady state. This completes the aggregation of the model.

The fiscal authority can levy lump-sum taxes/subsidies, τ_t^L, as well as distortionary income taxes/subsidies, τ_t^D. Assume that fiscal policy is passive in the sense that the government's intertemporal budget constraint is satisfied independently of the value of the price level.

A competitive equilibrium is a set of processes c_t, h_t, mc_t, k_{t+1}, i_t, s_t, and $\widetilde{P}_t$ that satisfy

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \left(1 - \tau_t^D\right)\mathrm{mc}_t z_t F_h(k_t, h_t), \tag{44}$$

$$U_c(c_t, h_t) = \beta E_t U_c(c_{t+1}, h_{t+1})\left[\left(1 - \tau_{t+1}^D\right)\mathrm{mc}_{t+1} z_{t+1} F_k(k_{t+1}, h_{t+1}) + (1-\delta)\right], \tag{45}$$

$$k_{t+1} = (1-\delta)k_t - i_t, \tag{46}$$

$$\frac{1}{s_t}\left[z_t F(k_t, h_t) - \chi\right] = c_t + i_t, \tag{47}$$

$$s_t = (1-\alpha)\widetilde{p}_t^{\,-\eta} + \alpha\pi_t^{\eta} s_{t-1}, \tag{48}$$

$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\widetilde{p}_t^{\,1-\eta}, \tag{49}$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_c(c_s, h_s)}{U_c(c_t, h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right)^{-\eta} (c_s + i_s)\left[\mathrm{mc}_s - \left(\frac{\eta - 1}{\eta}\right)\left(\widetilde{p}_t \prod_{k=t+1}^{s} \pi_k^{-1}\right)\right] = 0, \tag{50}$$

given the policy processes τ_t^D and π_t, the exogenous process z_t, and the initial conditions k_0 and s_{-1}. We assume that $s_{-1} = 1$.[6]

6.2 Optimality of zero inflation with production subsidies

We now show that the optimal monetary policy calls for price stability at all times. To see this, set $\pi_t = 1$ and $\tau_t^D = -\frac{1}{\eta-1}$ for all $t \geq 0$. It follows from equilibrium condition (49) that $\widetilde{P}_t = 1$ at all times and from Eq. (48) that $s_t = 1$ for all $t \geq 0$ as well. Now consider the conjecture $\text{mc}_t = (\eta - 1)/\eta$ for all $t \geq 0$. Under this conjecture equilibrium condition (50) is satisfied for all t. The remaining equilibrium conditions, (44)–(47), then simplify to

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = z_t F_h(k_t, h_t),$$

$$U_c(c_t, h_t) = \beta E_t U_c(c_{t+1}, h_{t+1})[z_{t+1} F_k(k_{t+1}, h_{t+1}) + (1-\delta)],$$

$$z_t F(k_t, h_t) - \chi = c_t + k_{t+1} - (1-\delta)k_t.$$

This is a system of three equations in the three unknowns, c_t, h_t, k_{t+1}. Note that these equations are identical to the optimality conditions of the social planner problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

subject to

$$z_t F(k_t, h_t) - \chi = c_t + k_{t+1} - (1-\delta)k_t.$$

We have therefore demonstrated that the policy $\pi_t = 1$ and $\left(1 - \tau_t^D\right) = \eta/(\eta - 1)$ induces a competitive-equilibrium real allocation that is identical to the real allocation associated with the social planner's problem. Therefore the proposed policy is not only Ramsey optimal but also Pareto optimal.

It is remarkable that even though this economy is stochastic, the optimal policy regime calls for deterministic paths of the aggregate price level P_t and the income tax rate τ_t^D. Zero inflation is the optimal monetary policy in the context of this model because it eliminates the relative price dispersion that arises when firms change prices in a staggered fashion. The proposed policy creates an environment in which firms never wish (even in the presence of uncertainty) to change the nominal price of the good they sell. We note that under the optimal policy τ_t^D is time invariant and negative (recall that $\eta > 1$).

[6] This assumption eliminates transitional dynamics in the Ramsey equilibrium. For a study of optimal policy in the case that this assumption is not satisfied see Yun (2005).

The negativity of τ_t^D implies that the Ramsey government subsidizes the use of capital and labor services to raise output above the level associated with the imperfectly competitive equilibrium and up to the level that would arise in a perfectly competitive equilibrium in which each intermediate goods-producing firm is compensated in a lump-sum fashion for its sunk cost χ.

The assumption that the government can subsidize factor inputs and finance such subsidies with lump-sum taxation is perhaps not the most compelling one. And it is therefore of interest to ask whether the optimality of zero inflation at all times continues to be true when it is assumed that the government does not have access to a subsidy. We consider this case in the next subsection.

6.3 Optimality of zero inflation without production subsidies

In this subsection, we investigate whether the optimality of zero inflation is robust to assuming that the government lacks access to the subsidy τ_t^D. We show analytically that in the Ramsey steady state the inflation rate is zero. That is, the Ramsey planner does not use inflation to correct distortions stemming from monopolistic competition. Although the proof of this result is somewhat tedious, we provide it here because to our knowledge it does not exist elsewhere in the literature.[7]

We begin by writing the first-order condition (50) recursively. To this end we introduce two auxiliary variables, x_t^1 and x_t^2, which denote an output weighted present discounted value of marginal revenues and marginal costs, respectively. Formally, we write Eq. (50) as

$$x_t^1 = x_t^2 \tag{51}$$

where

$$x_t^1 \equiv E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \frac{U_c(c_s, h_s)}{U_c(c_t, h_t)} \tilde{p}_t^{1-\eta} \left(\frac{P_t}{P_s}\right)^{1-\eta} (c_s + i_s) \left(\frac{\eta - 1}{\eta}\right)$$

and

$$x_t^2 \equiv E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \frac{U_c(c_s, h_s)}{U_c(c_t, h_t)} \tilde{p}_t^{-\eta} \left(\frac{P_t}{P_s}\right)^{-\eta} (c_s + i_s) \mathrm{mc}_s.$$

[7] Benigno and Woodford (2005) proved the optimality of zero steady-state inflation in the context a of Calvo-Yun-type sticky-price model without capital, with particular functional forms for the production and the utility functions, and with firm-specific labor. King and Wolman (1999) showed the optimality of zero steady-state inflation in the context of a sticky-price model with two-period Taylor-type price staggering, no capital, linear technology, and a specific period utility function.

The variables x_t^1 and x_t^2 can be written recursively as

$$x_t^1 = \widetilde{p}_t^{1-\eta}(c_t + i_t)\left(\frac{\eta - 1}{\eta}\right) + \alpha\beta E_t\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{1-\eta}\pi_{t+1}^{\eta-1}\frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)}x_{t+1}^1 \tag{52}$$

and

$$x_t^2 = \widetilde{p}_t^{\,-\eta}(c_t + i_t)mc_t + \alpha\beta E_t\frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)}\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{-\eta}\pi_{t+1}^{\eta}x_{t+1}^2. \tag{53}$$

The Ramsey planner then chooses c_t, h_t, mc_t, k_{t+1}, i_t, s_t, π_t, c_t, h_t, mc_t, k_{t+1}, i_t, s_t, π_t, x_t^1, x_t^2, and $\widetilde{p}_t$ and $\widetilde{P}_t$ to maximize Eq. (1) subject to Eqs. (40), (44), (45), (47), (48), (49), (51), (52), and (53) with $\tau_t^D = 0$ at all times and given the exogenous process z_t and the initial conditions k_0 and s_{-1}.

We are particularly interested in deriving the first-order conditions of the Ramsey problem with respect to π_t, $\widetilde{p}_t$, and x_t^1. Letting λ_t^1 denote the Lagrange multiplier on Eq. (52), λ_t^2 the multiplier on Eq. (53), λ_t^3 the multiplier on Eq. (49), and λ_t^4 the multiplier on Eq. (48), the part of the Lagrangian of the Ramsey problem that is relevant for our purpose (i.e., the part that contains π_t, $\widetilde{P}_t$, and x_t^1) is the following

$$\begin{aligned} L = \sum_{t=0}^{\infty}\beta^t\Bigg\{ \cdots &+ \lambda_t^1\left[\widetilde{p}_t^{1-\eta}(c_t+i_t)\left(\frac{\eta-1}{\eta}\right) + \alpha\beta E_t\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{1-\eta}\pi_{t+1}^{\eta-1}\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}x_{t+1}^1 - x_t^1\right] \\ &+ \lambda_t^2\left[\widetilde{p}_t^{\,-\eta}(c_t+i_t)mc_t + \alpha\beta E_t\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{-\eta}\pi_{t+1}^{\eta}x_{t+1}^1 - x_t^1\right] \\ &+ \lambda_t^3\left[\alpha\pi_t^{\eta-1} + (1+\alpha)\widetilde{p}_t^{1-\eta} - 1\right] + \lambda_t^4\left[(1-\alpha)\widetilde{p}_t^{\,-\eta} + \alpha\pi_t^{\eta}s_{t-1} - s_t\right] + \ldots \Bigg\} \end{aligned}$$

where we have replaced x_t^2 with x_t^1. The first-order conditions with respect to π_t, $\widetilde{P}_t$, and x_t^1, in that order, are

$$\begin{aligned} &\lambda_{t-1}^1\left[\alpha\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{1-\eta}\pi_t^{\eta-2}(\eta-1)\frac{U_c(c_t,h_t)}{U_c(c_{t-1},h_{t-1})}x_t^1\right] + \lambda_{t-1}^2\left[\eta\alpha\frac{U_c(c_t,h_t)}{U_c(c_{t-1},h_{t-1})}\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{-\eta}\pi_t^{\eta-1}x_t^1\right] \\ &\quad + \lambda_t^3\left[(\eta-1)\alpha\pi_t^{\eta-2}\right] + \lambda_t^4\left[\eta\alpha\pi_t^{\eta-1}s_{t-1}\right] = 0 \end{aligned}$$

$$\lambda_t^1(1-\eta)\widetilde{p}_t^{-\eta}(c_t+i_t)\left(\frac{\eta-1}{\eta}\right)$$
$$+\lambda_{t-1}^1\alpha(\eta-1)(1/\widetilde{p}_t)\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{1-\eta}\pi_t^{\eta-1}\frac{U_c(c_t,h_t)}{U_c(c_{t-1},h_{t-1})}x_t^1$$
$$+\lambda_t^1\left[+\alpha\beta(1-\eta)(1/\widetilde{p}_t)\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{1-\eta}\pi_{t+1}^{\eta-1}\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}x_{t+1}^1\right]$$
$$\lambda_t^2(-\eta)\widetilde{p}_t^{-\eta-1}(c_t+i_t)mc_t$$
$$+\lambda_{t-1}^2\alpha(\eta)(1/\widetilde{p}_t)\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{-\eta}\pi_{t+1}^{\eta}\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}x_{t+1}^1$$
$$+\lambda_t^2\left[+\alpha\beta(-\eta)(1/\widetilde{p}_t)\left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}\right)^{-\eta}\pi_{t+1}^{\eta}\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}x_{t+1}^1\right]$$
$$+\lambda_t^3(1-\alpha)(1-\eta)\widetilde{p}_t^{-\eta}+\lambda_t^4(1-\alpha)(-\eta)\widetilde{p}_t^{-\eta-1}=0$$

$$-\lambda_t^1+\lambda_{t-1}^1\alpha\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{1-\eta}\pi_t^{\eta-1}\frac{U_c(c_t,h_t)}{U_c(c_{t-1},h_{t-1})}-\lambda_t^2+\lambda_{t-1}^2\alpha\left(\frac{\widetilde{p}_{t-1}}{\widetilde{p}_t}\right)^{-\eta}\pi_t^{\eta}\frac{U_c(c_t,h_t)}{U_c(c_{t-1},h_{t-1})}=0$$

We restrict attention to the Ramsey steady state and thus can drop all time subscripts. We want to check whether a Ramsey steady state with $\pi=1$ exists. Given a value for π, we can find $\widetilde{p}$, k, c, h, i, x^1, s, and mc from the competitive equilibrium conditions (40), (44), (45), (47), (48), (49), (51), (52), and (53) and imposing $\tau_t^D=0$. Specifically, when $\pi=1$ by Eq. (49) we have that $\widetilde{p}$ 1, by Eq. (48) that $s=1$, and by Eqs. (51), (52), and (53) that $(\eta-1)/\eta=$ mc. We can then write the steady-state version of the preceding three first-order conditions as

$$\lambda^1[\alpha(\eta-1)x^1]+\lambda^2[\eta\alpha x^1]+\lambda^3(\eta-1)\alpha+\lambda^4\eta\alpha=0 \tag{54}$$

$$\lambda^1(1-\eta)(1-\alpha)x^1-\eta\lambda^2(1-\alpha)x^1+\lambda^3(1-\alpha)(1-\eta)+\lambda^4(1-\alpha)(-\eta)=0 \tag{55}$$

and

$$\lambda^1=\lambda^2=0.$$

Replacing λ^2 by $-\lambda^1$ and collecting terms, Eqs. (54) and (55) become the same expression, namely,

$$-\lambda^1x^1+\lambda^3(\eta-1)+\lambda^4\eta=0.$$

At this point, under the proposed solution $\pi=1$, we have in hand steady-state values for π, $\widetilde{p}$, s, mc, x^1, k, i, c, h, and two restrictions on Lagrange multipliers; namely,

$\lambda^2 = -\lambda^1$ and $\lambda^1 = (\eta\lambda^4 + (\eta - 1)\lambda^3)/x^1$. This leaves six Lagrange multipliers, which are λ^3 through λ^8, to be determined. We have not used yet the first-order conditions with respect to s_t, mc_t, k_{t+1}, i_t, c_t, and h_t, which are six linear equations in the remaining six Lagrange multipliers. We therefore have shown that $\pi = 1$ is a solution to the first-order conditions of the Ramsey problem in steady state. The key step in this proof was to show that when $\pi = 1$, first-order conditions (54) and (55) are not independent equations.

The optimality of zero inflation in the absence of production subsidies extends to the case with uncertainty. In Schmitt-Grohé and Uribe (2007a), we show numerically in the context of a production economy with capital accumulation like the one presented here, that even outside of the steady state the inflation rate is for all practical purposes equal to zero at all times. Specifically, Schmitt-Grohé and Uribe (2007a) find that for plausible calibrations the Ramsey optimal standard deviation of inflation is only 3 basis points at an annual rate.

6.4 Indexation

Thus far, we have assumed that firms that cannot reoptimize their prices in any given period simply maintain the price charged in the previous period. We now analyze whether the optimal rate of inflation would be affected if one assumed instead that firms follow some indexation scheme in their pricing behavior. A commonly studied indexation scheme is one where nonreoptimized prices increase mechanically at a rate proportional to the economy-wide lagged rate of inflation. Formally, under this indexation mechanism, any firm i that cannot reoptimize its price in period t sets $P_{it} = P_{it-1}\pi_{t-1}^{\iota}$, where $\iota \in [0, 1]$, is a parameter measuring the degree of indexation. When ι equals zero, the economy exhibits no indexation, which is the case we have studied thus far. When ι equals unity, prices are fully indexed to past inflation. And in the intermediate case in which ι lies strictly between zero and one, the economy is characterized by partial price indexation.

Consider the sticky-price economy with a production subsidy studied in Section 6.1 augmented with an indexation scheme like the one described in the previous paragraph. The set of equilibrium conditions associated with the indexed economy is identical to that of the economy of Section 6.1, with the exception that Eqs. (48)–(50) are replaced by

$$s_t = (1 - \alpha)\widetilde{p}_t^{\,-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\iota}}\right)^{\eta} s_{t-1}, \tag{56}$$

$$1 = \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\iota}}\right)^{\eta-1} + (1 - \alpha)\widetilde{p}_t^{\,1-\eta} \tag{57}$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_c(c_s, h_s)}{U_c(c_t, h_t)} (c_s + i_s) \left(\prod_{k=t+1}^{s} \frac{\pi_k}{\pi_{k-1}^{\iota}} \right)^{\eta} \left[\mathrm{mc}_s - \left(\frac{\eta - 1}{\eta} \right) \left(\widetilde{p}_t \prod_{k=t+1}^{s} \frac{\pi_{k-1}^{\iota}}{\pi_k} \right) \right] = 0. \tag{58}$$

We continue to assume that $s_{-1} = 1$. Note that when $\iota = 0$, these three expressions collapse to Eqs. (48)–(50). This means that the model with indexation nests the model without indexation as a special case. For any $\iota \in [0, 1]$, the Ramsey optimal policy is to set $\pi_t = \pi_{t-1}^{\iota}$ for all $t \geq 0$. To see this, note that under this policy the solution to the previous three equilibrium conditions is given by $\widetilde{P}_t = 1$, $s_t = 1$, and $\mathrm{mc}_t = (\eta - 1)/\eta$ for all $t \geq 0$. Then, recalling that we are assuming the existence of a production subsidy τ_t^D equal to $-1/(\eta - 1)$ at all times and by the same logic applied in Section 6.2, the remaining equilibrium conditions of the model, given by Eqs. (44)–(47), collapse to the optimality conditions of an economy with perfect competition and flexible prices. It follows that the proposed policy is both Ramsey optimal and Pareto efficient. The intuition behind this result is simple. By inducing firms that can reoptimize prices to voluntarily mimic the price adjustment of firms that cannot reoptimize, the policymaker ensures the absence of price dispersion across firms.

In the case of partial indexation, that is, when $\iota < 1$, the Ramsey optimal rate of inflation converges to zero. So, under partial indexation, just as in the case of no indexation studied in previous sections, the Ramsey steady state features zero inflation. When the inherited inflation rate is different from zero ($\pi_{-1} \neq 1$), the convergence of inflation to zero is gradual under the optimal policy. The speed of convergence to price stability is governed by the parameter ι. This feature of optimal policy has an important implication for the design of inflation stabilization strategies in countries in which the regulatory system imposes an exogenous indexation mechanism on prices (such as Chile in the 1970s and Brazil in the 1980s). The results derived here suggest that in exogenously indexed economies it would be suboptimal to follow a cold turkey approach to inflation stabilization. Instead, in this type of economies, policymakers are better advised to follow a gradualist approach to inflation stabilization, or, alternatively, to dismantle the built-in indexation mechanism before engaging in radical inflation reduction efforts. A different situation arises when the indexation mechanism is endogenous, instead of imposed by regulation. Endogenous indexation naturally arises in economies undergoing high or hyperinflation. In this case, a cold turkey approach to disinflation is viable because agents will relinquish their indexation schemes as inflationary expectations drop.

Consider now the polar case of full indexation, or $\iota = 1$. In this case the monetary policy that is both Ramsey optimal and Pareto efficient is to set π_t equal to π_{-1} at all times. That is, under full indexation, the optimal monetary policy in the short and long runs is determined by the country's inflationary history. Empirical studies of the degree of price indexation for the United States do not support the assumption of full indexation, however. For example, the econometric estimates of the degree of price indexation reported by Cogley and Sbordone (2008) and Levin, Onatski, Williams, and Williams (2006), in the context of models exhibiting Calvo-Yun price staggering, concentrate around zero. We therefore conclude that for plausible parameterizations of the Calvo-Yun sticky-price model, the Ramsey optimal inflation rate in the steady state is zero.

7. THE FRIEDMAN RULE VERSUS PRICE-STABILITY TRADE-OFF

We have established thus far that in an economy in which the only nominal friction is a demand for fiat money, deflation at the real rate of interest (the Friedman rule) is optimal. We have also shown that when the only nominal friction is the presence of nominal-price-adjustment costs, zero inflation emerges as the Ramsey optimal monetary policy. A realistic economic model, however, should incorporate both a money demand and price stickiness. In such an environment, the Ramsey planner faces a tension between minimizing the opportunity cost of holding money and minimizing the cost of price adjustments. One would naturally expect, therefore, that when both the money demand and the sticky-price frictions are present, the optimal rate of inflation falls between zero and the one called for by the Friedman rule. The question of interest, however, is where exactly in this interval the optimal inflation rate lies. No analytical results are available on the resolution of this trade-off. We therefore carry out a numerical analysis of this issue. The resolution of the Friedman-rule-versus-price-stability trade-off has been studied in Khan, King, and Wolman (2003) and in Schmitt-Grohé and Uribe (2004a, 2007b).

To analyze the Friedman-rule-versus-price-stability trade-off, we augment the sticky-price model of Section 6 with a demand for money like the one introduced in Section 2. That is, in the model of the previous section we now assume that consumers face a transaction cost $s(v_t)$ per unit of consumption, where $v_t \equiv c_t P_t / M_t$ denotes the consumption-based velocity of money. A competitive equilibrium in the economy with sticky prices and a demand for money is a set of processes c_t, v_t, h_t, mc_t, k_{t+1}, i_t, s_t, $\widetilde{P}_t$, and π_t that satisfy

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{\left(1-\tau_t^D\right)\mathrm{mc}_t z_t F_h(k_t, h_t)}{1+s(v_t)+v_t s'(v_t)},$$

$$\frac{U_c(c_t, h_t)}{1+s(v_t)+v_t s'(v_t)} = \beta E_t \frac{U_c(c_{t+1}, h_{t+1})}{1+s(v_{t+1})+v_{t+1}s'(v_{t+1})}\left[\left(1-\tau_{t+1}^D\right)\mathrm{mc}_{t+1} z_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta\left(1-\tau_{t+1}^D\right)\right], \tag{59}$$

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{60}$$

$$\frac{1}{s_t}[z_t F(k_t, h_t) - \chi] = c_t[1 + s(v_t)] + i_t, \tag{61}$$

$$s_t = (1-\alpha)\widetilde{p}_t^{\,-\eta} + \alpha\pi_t^{\eta} s_{t-1}, \tag{62}$$

$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\widetilde{p}_t^{1-\eta}, \tag{63}$$

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_c(c_s, h_s)}{U_c(c_t, h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1} \right)^{-\eta} \{c_s[1+s(v_s)] = i_s\} \left[\mathrm{mc}_s - \left(\frac{\eta-1}{\eta}\right) \left(\widetilde{p}_t \prod_{k=t+1}^{s} \pi_k^{-1} \right) \right] = 0, \tag{64}$$

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t},$$

and

$$\frac{U_c(c_t, h_t)}{1 + s(v_t) + v_t s'(v_t)} = \beta R_t E_t \frac{U_c(c_{t+1}, h_{t+1})}{1 + s(v_{t+1}) + v_{t+1} s'(v_{t+1})} \frac{1}{\pi_{t+1}},$$

given the policy processes τ_t^D and R_t, the exogenous process z_t, and the initial conditions k_0 and s_{-1}.

We begin by considering the case in which the government has access to lump-sum taxes. Therefore, we set τ_t^D equal to zero for all t. We assume that the utility function is of the form given in Eq. (18) and that the production technology is of the form $F(k, h) = k^{\omega} h^{1-\omega}$, with $\omega \in (0,1)$. The transaction cost technology takes the form given in Eq. (20). We assume that the time unit is a quarter and calibrate the structural parameters of the model as follows: $A = 0.22$, $B = 0.13$, $\theta = 1.1$, $\omega = 0.36$, $\delta = 0.025$, $\beta = 0.9926$, $\eta = 6$, $\chi = 0.287$, and $\alpha = 0.8$. We set the parameter χ so that profits are zero. The calibrated values of A and B imply that at a nominal interest rate of 5.5% per year, which is the mean 3-month Treasury Bill rate observed in the United States between 1966:Q1 and 2006:Q4, the implied money-to-consumption ratio is 31% per year, which is in line with the average M1-to-consumption ratio observed in the United States over the same period. The calibrated value of α of 0.8 implies that prices have an average duration of 5 quarters. We focus on the steady state of the Ramsey optimal competitive equilibrium.

Note that the Ramsey steady state is generally different from the allocation/policy that maximizes welfare in the steady state of a competitive equilibrium. We apply the numerical algorithm developed in Schmitt-Grohé and Uribe (2006), which calculates the exact value of the Ramsey steady state. We find that the optimal rate of inflation is −0.57% per year. As expected, the Ramsey optimal inflation rate falls between the one called for by the Friedman rule, which under our calibration is

-2.91% per year, and the one that is optimal when the only nominal friction is price stickiness, which is an inflation rate of 0%. Our calculations show, however, that the optimal rate of inflation falls much closer to the inflation rate that is optimal in a cashless economy with sticky prices than to the inflation rate that is optimal in a monetary economy with flexible prices. This finding suggests that the Friedman rule versus sticky-price trade-off is resolved in favor of price stability. We now study the sensitivity of this finding to changes in three key structural parameters of the model. One parameter is α, which determines the degree of price stickiness. The second parameter is B, which pertains to the transactions cost technology and determines the interest elasticity of money demand. The third parameter is A, which also belongs to the transaction cost function and governs the share of money in output

7.1 Sensitivity of the optimal rate of inflation to the degree of price stickiness

Schmitt-Grohé and Uribe (2007b) found that a striking characteristic of the optimal monetary regime is the high sensitivity of the welfare-maximizing rate of inflation with respect to the parameter α, governing the degree of price stickiness, for the range of values of this parameter that is empirically relevant.

The parameter α measures the probability that a firm is not able to optimally set the price it charges in a particular quarter. The average number of periods elapsed between two consecutive optimal price adjustments is given by $1/(1-\alpha)$. Available empirical estimates of the degree of price rigidity using macroeconomic data vary from 2 to 6.5 quarters, or $\alpha \in [0.5, 0.85]$. For example, Christiano, Eichenbaum, and Evans (2005) estimated α to be 0.6. By contrast, Altig, Christiano, Eichenbaum, and Lindé (2005) estimated a marginal-cost-gap coefficient in the Phillips curve that is consistent with a value of α of around 0.8. Both Christiano et al. (2005) and Altig et al. (2005) used an impulse response matching technique to estimate the price-stickiness parameter α. Bayesian estimates of this parameter include Del Negro, Schorfheide, Smets, and Wouters (2004); Levin et al. (2006); and Smets and Wouters (2007) who reported posterior means of 0.67, 0.83, and 0.66, respectively, and 90% posterior probability intervals of (0.51,0.83), (0.81,0.86), and (0.56,0.74), respectively.

Recent empirical studies have documented the frequency of price changes using micro data underlying the construction of the U.S. consumer price index. These studies differ in the sample period considered, in the disaggregation of the price data, and in the treatment of sales and stockouts. The median frequency of price changes reported by Bils and Klenow (2004) is 4 to 5 months, the one reported by Klenow and Kryvtsov (2005) is 4 to 7 months, and the one reported by Nakamura and Steinsson (2007) is 8 to 11 months. However, there is no immediate translation of these frequency estimates to the parameter α governing the degree of price stickiness in Calvo-style models of price staggering. Consider, for instance, the case of indexation. In the presence of

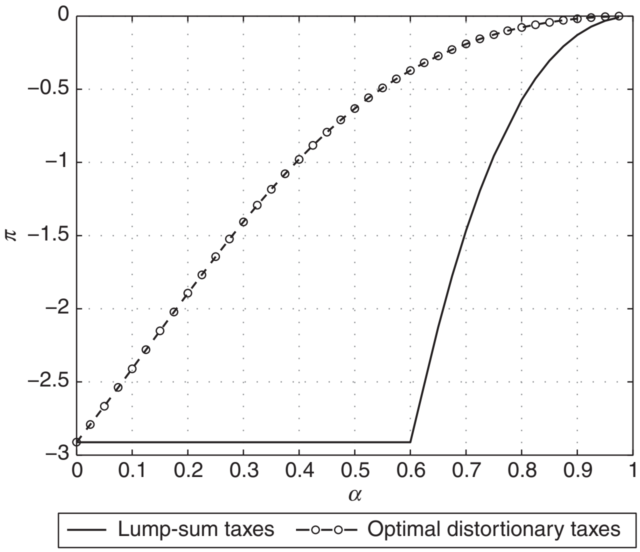

Figure 1 Price stickiness, fiscal policy, and optimal inflation.

indexation, even though firms change prices every period — implying the highest possible frequency of price changes — prices themselves may be highly sticky for they may only be reoptimized at much lower frequencies.

Figure 1 displays with a solid line the relationship between the degree of price stickiness, α, and the optimal rate of inflation in percent per year, π, implied by the model under study. When α equals 0.5, the lower range of the available empirical evidence using macro data, the optimal rate of inflation is -2.9%, which is the level called for by the Friedman rule. For a value of α of 0.85, which is near the upper range of the available empirical evidence using macro data, the optimal level of inflation rises to -0.3, which is close to price stability.

This finding suggests that given the uncertainty surrounding the empirical estimates of the degree of price stickiness, the neo-Keynesian model studied here does not deliver a clear recommendation regarding the level of inflation that a benevolent central bank should target. This difficulty is related to the shape of the relationship linking the degree of price stickiness to the optimal level of inflation. The problem resides in the fact that, as is evident from Figure 1, this relationship becomes significantly steep precisely for the range of values of α that is empirically most compelling.

It turns out that an important factor determining the shape of the function relating the optimal level of inflation to the degree of price stickiness is the underlying fiscal policy regime. Schmitt-Grohé and Uribe (2007b) showed that fiscal considerations fundamentally change the long-run trade-off between price stability and the Friedman rule. To see this, we now consider an economy where lump-sum taxes are unavailable. Instead, the fiscal authority must finance its budget by means of proportional income taxes. Formally, in this specification of the model, the Ramsey planner sets optimally not only the monetary policy instrument, R_t, but also the fiscal policy instrument, τ_t^D. Figure 1 displays with a dash-circled line the relationship between the degree of price stickiness, α, and the optimal rate of inflation, π, in the economy with optimally chosen fiscal and monetary policy. In stark contrast to what happens under lump-sum taxation, under optimal distortionary income taxation the function linking π and α is flat and close to zero for the entire range of macro-data-based empirically plausible values of α, namely 0.5 to 0.85. In other words, when taxes are distortionary and optimally determined, price stability emerges as a prediction that is robust to the existing uncertainty about the exact degree of price stickiness.

Our intuition for why price stability arises as a robust policy recommendation in the economy with optimally set distortionary taxation runs as follows. Consider the economy with lump-sum taxation. Deviating from the Friedman rule (by raising the inflation rate) has the benefit of reducing price adjustment costs. Consider next the economy with optimally chosen income taxation and no lump-sum taxes. In this economy, deviating from the Friedman rule still provides the benefit of reducing price adjustment costs. However, in this economy increasing inflation has the additional benefit of increasing seignorage revenue, allowing the social planner to lower distortionary income tax rates. Therefore, the Friedman-rule versus price-stability trade-off is tilted in favor of price stability.

It follows from this intuition that what is essential in inducing the optimality of price stability is that on the margin the fiscal authority trades off the inflation tax for regular taxation. Indeed, it can be shown that if distortionary tax rates are fixed, even if they are fixed at the level that is optimal in a world without lump-sum taxes, and the fiscal authority has access to lump-sum taxes on the margin, the optimal rate of inflation is much closer to the Friedman rule than to zero. In this case, increasing inflation no longer has the benefit of reducing distortionary taxes. As a result, the Ramsey planner has less incentives to inflate (see Schmitt-Grohé & Uribe, 2007b).

It is remarkable that in a flexible-price, monetary economy the optimal rate of inflation is insensitive to whether the government has access to distortionary taxation or not. In effect, we have seen that in a flexible-price environment with a demand for money it is always optimal to set the inflation rate at the level called for by the Friedman rule. Indeed, this characteristic of optimal policy in the flexible price model led an entire literature in the 1990s to dismiss Phelps' (1973) conjecture that the presence of distortionary taxes should induce a departure from the Friedman rule. This conjecture, however, regains validity when evaluated in the context of models with price rigidities. As is evident from our discussion of Figure 1 in a monetary economy

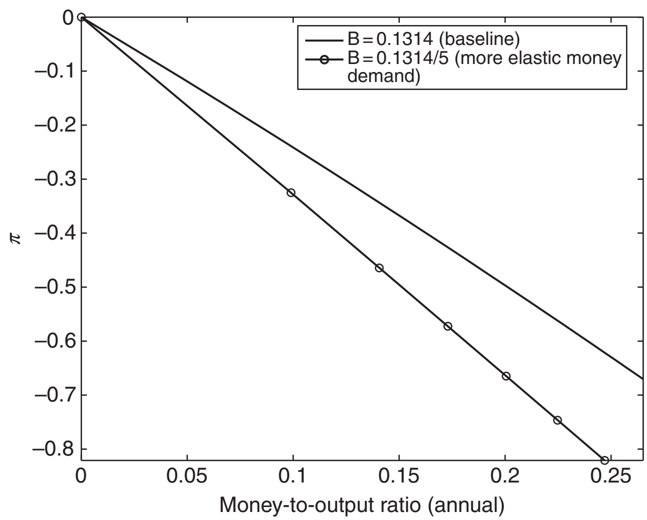

Figure 2 The optimal inflation rate as a function of the money-to-output share in the sticky-price model.

with price stickiness, the optimal rate of inflation is highly sensitive to the type of fiscal instrument available to the government.

7.2 Sensitivity of the optimal rate of inflation to the size and elasticity of money demand

Figure 2 displays the steady-state Ramsey optimal rate of inflation as a function of the share of money in output in the model with lump-sum taxes. The range of money-to-output ratios on the horizontal axis of the figure is generated by varying the parameter A in the transactions cost function from 0 to 0.3. The special case of a cashless economy corresponds to the point in the figure in which the share of money in output equals zero (that is, $A = 0$). Figure 2 shows that at this point the Ramsey optimal rate of inflation is equal to zero. This result demonstrates that even in the absence of production subsidies aimed at eliminating the inefficiency associated with imperfect competition in product markets (recall that we are assuming that $\tau_t^D = 0$), the optimal rate of inflation is zero when the only source of nominal frictions is the presence of sluggish price adjustment. This result numerically illustrates the one obtained analytically in Section 6.3.

Figure 2 shows that as the value of the parameter A increases, the money-to-output share rises and the Ramsey optimal rate of inflation falls. This is because when the demand for money is nonzero, the social planner must compromise between price stability (which minimizes the costs of nominal price dispersion across intermediate-good producing firms) and deflation at the real rate of interest (which minimizes the opportunity cost of holding money). This figure shows that even at money-to-output ratios as high as 25%, the optimal rate of inflation is far above the one called for by the Friedman rule (−0.65%vs. −2.9%, respectively).

Under our baseline calibration the implied money demand elasticity is low. At a nominal interest rate of 0, the money-to-consumption ratio is only 2 percentage points higher than at a nominal interest rate of 5.5%. For this reason, we also consider a calibration in which the parameter B of the transaction cost function is five times smaller and adjust the parameter A so that money demand continues to be 31% of consumption at an annual interest rate of 5.5%. Under this alternative calibration, money demand increases from 31 to 40% as the interest rate falls from the average U.S. value of 5.5% to 0%. The relationship between the share of money in output and the optimal rate of inflation in the economy with the high interest elasticity of money demand is shown with a circled line in Figure 2. It shows that even when the interest elasticity is five times higher than in the baseline case, the optimal rate of inflation remains near zero. Specifically, the largest decline in the optimal rate of inflation occurs at the high end of money-to-output ratios considered and is only 15 basis points. We conclude that for plausible calibrations the price-stickiness friction dominates the optimal choice of long-run inflation.

Note: In the baseline case, the range of values of the money-to-output ratio is obtained by varying the parameter A of the transaction cost function from 0 to 0.3 and keeping all other parameters of the model constant.

We wish to close this section by drawing attention to the fact that, quite independently of the precise degree of price stickiness or the size and elasticity of money demand, the optimal inflation target is at most zero. In light of this robust result, it remains hard to rationalize why countries that self-classify as inflation targeters set inflation targets that are positive. An argument often raised in defense of positive inflation targets is that negative inflation targets imply nominal interest rates that are dangerously close to the zero lower bound on nominal interest rates and hence may impair the central bank's ability to conduct stabilization policy. We will evaluate the merits of this argument in the following section.

8. DOES THE ZERO BOUND PROVIDE A RATIONALE FOR POSITIVE INFLATION TARGETS?

One popular argument against setting a zero or negative inflation target is that at zero or negative rates of inflation the risk of hitting the zero lower bound on nominal interest rates would severely restrict the central bank's ability to conduct successful stabilization policy. This argument is made explicit, for example, in Summers (1991). The evaluation of this argument hinges critically on assessing how frequently the zero bound would be hit under optimal policy. It is therefore a question that depends primarily on the size of exogenous shocks the economy is subject to and on the real and nominal frictions that govern the transmission of such shocks. We believe therefore that this argument is best evaluated in the context of an empirically realistic quantitative model of the business cycle. In Schmitt-Grohé and Uribe (2007b) we study Ramsey optimal monetary policy in an estimated medium-scale model of the macroeconomy. The theoretical framework employed there emphasizes the importance of combining nominal as well as real rigidities in explaining the propagation of macroeconomic shocks. Specifically, the model features four nominal frictions, sticky prices, sticky wages, a transactional demand for money by households, and a cash-in-advance constraint on the wage bill of firms, and four sources of real rigidities, investment adjustment costs, variable capacity utilization, habit formation, and imperfect competition in product and factor markets. Aggregate fluctuations are driven by three shocks: a permanent neutral labor-augmenting technology shock, a permanent investment-specific technology shock, and temporary variations in government spending. Altig et al. (2005) and Christiano et al. (2005), using a limited information econometric approach, argued that the model economy for which we seek to design optimal monetary policy can indeed explain the observed responses of inflation, real wages, nominal interest rates, money growth, output, investment, consumption, labor productivity, and real profits to neutral and investment-specific productivity shocks and monetary shocks in the post-war United States. Smets and Wouters (2003, 2007) also concluded, on the basis of a full information Bayesian econometric estimation, that the medium-scale neo-Keynesian framework provides an adequate framework for understanding business cycles in the post-war United States and Europe.

In the simulations reported in this section, we calibrate the three structural shocks as follows. We construct a time series of the relative price of investment in the United States for the period 1955Q1 to 2006Q4. We then use this time series to estimate an AR(1) process for the growth rate of the relative price of investment. The estimated serial correlation is 0.45 and the estimated standard deviation of the innovation of the process is 0.0037. These two figures imply that the growth rate of the price of investment has an unconditional standard deviation of 0.0042. Ravn (2005) estimated an AR(1) process for the detrended level of government purchases in the context of a model similar to the one we are studying and finds a serial correlation of 0.9 and a standard deviation of the innovation to the AR(1) process of 0.008. Finally, we assume that the permanent neutral labor-augmenting technology shock follows a random walk with a drift. We set the standard deviation of the innovation to this process at 0.0188, to match the observed volatility of per capita output growth of 0.91% per quarter in the United States over the period 1955Q1 to 2006Q4. For the purpose of calibrating this standard deviation, we assume that monetary policy takes the form of a Taylor-type interest rate feedback rule with an inflation coefficient of 1.5 and an output coefficient of 0.125. We note that in the context of our model an output coefficient of 0.125 in the interest rate feedback rule corresponds to the 0.5 output coefficient estimated by Taylor (1993). This is because Taylor estimates the interest rate feedback rule using annualized rates of interest and inflation whereas in our model these two rates are expressed in quarterly terms. All other parameters of the model are calibrated as in Schmitt-Grohé and Uribe (2007b). In particular, the subjective discount rate is set at 3% per year and the average growth rate of per-capita output at 1.8% per year. This means that in the deterministic steady state the real rate of interest equals 4.8%, a value common in business-cycle studies. After completing the calibration of the model, we drop the assumption that the monetary authority follows an interest rate feedback rule and proceed to characterize Ramsey optimal monetary policy ignoring the occasionally binding constraint implied by the zero bound.

The Ramsey optimal policy implies a mean inflation rate of −0.4% per year. This slightly negative inflation target is in line with the quantitative results we obtained in Section 7 using a much simpler model of the monetary transmission mechanism. More important for our purposes, however, are the predictions of the model for the Ramsey optimal level and volatility of the nominal rate of interest. Under the Ramsey optimal monetary policy, the standard deviation of the nominal interest rate is only 0.9 percentage points at an annual rate. At the same time, the mean of the Ramsey optimal level of the nominal interest rate is 4.4%. These two figures taken together imply that for the nominal interest rate to violate the zero bound, it must fall more than 4 standard deviations below its target level. This finding suggests that in the context of the model analyzed here, the probability that the Ramsey optimal nominal interest rate violates the zero bound is practically zero. This result is robust to lowering the deterministic real rate of interest. Lowering the subjective discount factor from its baseline value of 3 to 1% per year results in a Ramsey-optimal nominal interest rate process that has a mean of 2.4% per year and a standard deviation of 0.9% per year. This means that under this calibration the nominal interest rate must still fall by almost three standard deviations below its mean for the zero bound to be violated. Some have argued, however, that a realistic value of the subjective discount factor is likely to be higher and not lower than the value of 3% used in our baseline calibration. This argument arises typically from studies that set the discount factor to match the average risk-free interest rate in a nonlinear stochastic environment rather than simply to match the deterministic steady-state real interest rate (see, for instance, Campbell & Cochrane, 1999).

It is worth stressing that our analysis abstracted from the occasionally binding constraint imposed by the zero bound. However, the fact that in the Ramsey equilibrium the zero bound is violated so rarely leads us to conjecture that in an augmented version of the model that explicitly imposes the zero bound constraint, the optimal inflation target would be similar to the value of −0.4% per year that is optimal in the current model. This conjecture is supported by the work of Adam and Billi (2006). These authors computed the optimal monetary policy in a simpler version of the New Keynesian model considered in this section.

An advantage of their approach is that they take explicitly into account the zero bound restriction in computing the optimal policy regime. They find that the optimal monetary policy does not imply positive inflation on average and that the zero bound binds infrequently. Their finding of a nonpositive average optimal rate of inflation is, furthermore, of interest in light of the fact that their model does not incorporate a demand for money. We conjecture, based on the results reported in this section, that should a money demand be added to their framework, the average optimal rate of inflation would indeed be negative.

Reifschneider and Williams (2000) also considered the question of the optimal rate of inflation in the presence of the zero-lower-bound restriction on nominal rates. Their analysis is conducted within the context of the large-scale FRB/US model. In their exercise, the objective function of the central bank is to minimize a weighted sum of inflation and output square deviations from targets. They find that under optimized simple interest-rate feedback rules (which take the form of Taylor rules modified to past policy constraints or of Taylor rules that respond to the cumulative deviation of inflation from target) the zero bound has on average negligible effects on the central bank's ability to stabilize the economy. Further, these authors find that under optimized rules episodes in which the zero bound is binding are rare even at a low target rate of inflation of zero.

9. DOWNWARD NOMINAL RIGIDITY

One rationale for pursuing a positive inflation target that surfaces often in the academic and policy debate is the existence of asymmetries in nominal factor- or product-price rigidity. For instance, there is ample evidence suggesting that nominal wages are more rigid downward than upward (see, for instance, Akerlof, Dickens, and Perry, 1996; Card and Hyslop, 1997; and McLaughlin, 1994).

The idea that downward nominal price rigidity can make positive inflation desirable goes back at least to Olivera (1964), who referred to this phenomenon as structural inflation. The starting point of Olivera's analysis is a situation in which equilibrium relative prices are changed by an exogenous shock. In this context, and assuming that the monetary authority passively accommodates the required relative price change, Olivera explains the inflationary mechanism invoked by downward rigidity in nominal prices as follows:[8]

> *A clear-cut case is when money prices are only responsive to either positive or negative excess demand (unidirectional flexibility). Then every relative price adjustment gives rise to a variation of the price level, upward if there exists downward inflexibility of money prices, downward if*

[8] The model described in this passage is, as Olivera (1964) pointed out, essentially the same presented in his presidential address to the Argentine Association of Political Economy on October 8, 1959, and later published in Olivera (1960).

there is upward inflexibility. Thus, in a medium of downward inflexible money prices any adjustment of price-ratios reverberates as an increase of the money price-level (Olivera, 1964, p. 323.)

As for the desirability of inflation in the presence of nominal downward rigidities, Olivera (1964) wrote

As to the money supply, [. . .] the full-employment goal can be construed as requiring a pari passu adaptation of the financial base to the rise of the price-level [. . .]. (p. 326)

Clearly, Olivera's notion of "structural inflation" is tantamount to the metaphor of "inflation greasing the wheels of markets," employed in more recent expositions of the real effects of nominal downward rigidities. Tobin (1972) similarly argued that a positive rate of inflation may be necessary to avoid unemployment when nominal wages are downwardly rigid.

Kim and Ruge-Murcia (2009) quantified the effect of downward nominal wage rigidity on the optimal rate of inflation. They embedded downward nominal rigidity into a dynamic stochastic neo-Keynesian model with price stickiness and no capital accumulation. They modeled price and wage stickiness à la Rotemberg (1982). The novel element of their analysis is that wage adjustment costs are asymmetric. Specifically, the suppliers of differentiated labor inputs are assumed to be subject to wage adjustment costs, $\Phi(W_t^j/W_{t-1}^j)$, that take the form of a linex function in wage inflation:

$$\Phi\left(\frac{W_t^j}{W_{t-1}^j}\right) \equiv \phi\left[\frac{\exp\left(-\psi\left(W_t^j/W_{t-1}^j-1\right)\right)+\psi\left(W_t^j/W_{t-1}^j-1\right)-1}{\psi^2}\right]$$

where W_t^j denotes the nominal wage charged by supplier j in period t and φ and ψ are positive parameters. The wage-adjustment-cost function $\Phi(\cdot)$ is positive, strictly convex, and has a minimum of 0 at zero wage inflation $\left(W_t^j = W_{t-1}^j\right)$. More important, this function is asymmetric around zero wage inflation. Its slope is larger in absolute value for negative wage inflation rates than for positive ones. In this way, it captures the notion that nominal wages are more rigid downward than upward. As the parameter ψ approaches infinity, the function becomes L-shaped, corresponding to the limit case of full downward inflexibility and full upward flexibility. When ψ approaches zero, the adjustment cost function becomes quadratic, corresponding to the standard case of symmetric wage adjustment costs. Kim and Ruge-Murcia (2009) estimated the structural parameters of the model using a simulated method of moments technique and a second-order-accurate approximation of the model. They found a point estimate of the asymmetry parameter ψ of 3844.4 with a standard error of 1186.7.

The key result reported by Kim and Ruge-Murcia (2009) is that under the Ramsey optimal monetary policy the unconditional mean of the inflation rate is 0.35% per year. This figure is too small to explain the inflation targets of 2% observed in the industrial world. Moreover, this figure is likely to be an upper bound for the size of the inflation bias introduced by downward nominal rigidities in wages for the following two reasons. First, their model abstracts from a money-demand friction. It is expected that should such a friction be included in the model, the optimal rate of inflation would be smaller than the reported 35 basis points, as the policymaker would find it costly from the Friedman rule. Second, Kim and Ruge-Murcia's (2009) analysis abstracted from long-run growth in real wages. As these authors acknowledged, in a model driven only by aggregate disturbances, the larger the average growth rate of the economy, the less likely it is that real wages experience a decline over the business cycle; hence, that inflation is needed to facilitate the efficient adjustment of the real price of labor.

10. QUALITY BIAS AND THE OPTIMAL RATE OF INFLATION

In June 1995, the Senate Finance Committee appointed an advisory commission composed of five prominent economists (Michael Boskin, Ellen Dulberger, Robert Gordon, Zvi Griliches, and Dale Jorgenson) to study the magnitude of the measurement error in the consumer price index (CPI). The commission concluded that during 1995–1996, the U.S. CPI had an upward bias of 1.1% per year. Of the total bias, 0.6%was ascribed to unmeasured quality improvements. To illustrate the nature of the quality bias, consider the case of a personal computer. Suppose that between 1995 and 1996 the nominal price of a computer increased by 2%. Assume also that during this period the quality of personal computers, measured by attributes such as memory, processing speed, and video capabilities, increased significantly. If the statistical office in charge of producing the consumer price index did not adjust the price index for quality improvement, then it would report 2% inflation in personal computers. However, because a personal computer in 1996 provides more services than a personal computer from 1995, the quality-adjusted rate of inflation in personal computers should be recorded as lower than 2%. The difference between the reported rate of inflation and the quality-adjusted rate of inflation is called the quality bias in measured inflation.

The existence of a positive quality bias has led some to argue that an inflation target equal in size to the bias would be appropriate if the ultimate objective of the central bank is price stability. In this section, we critically evaluate this argument. Specifically, we study whether the central bank should adjust its inflation target to account for the systematic upward bias in measured inflation due to quality improvements in consumption goods. We show that the answer to this question depends critically on what prices are assumed to be sticky. If nonquality-adjusted prices are sticky, then the inflation target should not be corrected. If, on the other hand, quality-adjusted (or hedonic) prices are sticky, then the inflation target must be raised by the magnitude of the bias. Our analysis closely follows Schmitt-Grohé and Uribe (2009b).

10.1 A simple model of quality bias

We analyze the relationship between a quality bias in measured inflation and the optimal rate of inflation in the context of the neo-Keynesian model of Section 6.1 without capital. The key modification we introduce to that framework is that the quality of consumption goods is assumed to increase over time. This modification gives rise to an inflation bias if the statistical agency in charge of constructing the CPI fails to take quality improvements into account. The central question we entertain here is whether the inflation target should be adjusted by the presence of this bias.

The economy is populated by a large number of households with preferences defined over a continuum of goods of measure one indexed by $i \in [0,1]$. Each unit of good i sells for P_{it} dollars in period t. We denote the quantity of good i purchased by the representative consumer in period t by c_{it}. The quality of good i is denoted by x_{it} and is assumed to evolve exogenously and to satisfy $x_{it} > x_{it-1}$. The household cares about a composite good given by

$$\left[\int_0^1 (x_{it} c_{it})^{1-1/\eta} di\right]^{1/(1-1/\eta)},$$

where $\eta > 1$ denotes the elasticity of substitution across different good varieties. Note that the utility of the household increases with the quality content of each good. Let a_t denote the amount of the composite good the household wishes to consume in period t. Then, the demand for goods of variety i is the solution to the following cost-minimization problem

$$\min_{\{c_{it}\}} \int_0^1 P_{it} C_{it} \, di$$

subject to

$$\left[\int_0^1 (x_{it} C_{it})^{1-1/\eta} di\right]^{1/(1-1/\eta)} \geq a_t.$$

The demand for good i is then given by

$$C_{it} = \left(\frac{Q_{it}}{Q_t}\right)^{-\eta} \frac{a_t}{x_{it}},$$

where

$$Q_{it} \equiv P_{it}/x_{it}$$

denotes the quality-adjusted (or hedonic) price of good i, and Q_t is a quality-adjusted (or hedonic) price index given by

$$Q_t = \left[\int_0^1 Q_{it}^{1-\eta} di\right]^{1/(1-\eta)}.$$

The price index Q_t has the property that the total cost of a_t units of composite good is given by $Q_t a_t$, that is, $\int_0^1 P_{it} C_{it}\, di = Q_t a_t$. Because a_t is the object from which households derive utility, it follows from this property that Q_t, the unit price of a_t, represents the appropriate measure of the cost of living.

Households supply labor effort to the market for a nominal wage rate W_t and are assumed to have access to a complete set of financial assets. Their budget constraint is given by

$$Q_t a_t \; + \; E_t r_{t,t+1}\, D_{t+1} + \; T_t = D_t\; W_t h_t \; + \; \Phi_t,$$

where $r_{t,\ t+j}$ is a discount factor defined so that the dollar price in period t of any random nominal payment D_{t+j} in period $t + j$ is given by $E_t\, r_{t,t+j} D_{t+j}$. The variable Φ_t denotes nominal profits received from the ownership of firms, and the variable T_t denotes lump-sum taxes.

The lifetime utility function of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(a_t,\ h_t),$$

where the period utility function U is assumed to be strictly increasing and strictly concave and $\beta \in (0,\ 1)$. The household chooses processes $\{a_t,\ h_t,\ D_{t+1})$ to maximize this utility function subject to the sequential budget constraint and a no-Ponzi-game restriction of the form $\lim_{j\to\infty} E_t r_{t,t+j}\, D_{t+j} \;\geq\; 0$. The optimality conditions associated with the household's problem are the sequential budget constraint, the no-Ponzi-game restriction holding with equality, and

$$\frac{U_2(a_t,\ h_t)}{U_1(a_t,\ h_t)} = \frac{W_t}{Q_t}$$

and

$$\frac{U_1\ (a_t,\ h_t)}{Q_t}\ r_{t,t+1} = \beta \frac{U_1\ (a_{t+1},\ h_{t+1})}{Q_{t+1}}$$

Each intermediate consumption good $i \in [0,1]$ is produced by a monopolistically competitive firm via a linear production function $z_t h_{it}$, where h_{it} denotes labor input used in the production of good i, and z_t is an aggregate productivity shock. Profits of firm i in period t are given by

$$P_{it} C_{it} - W_t h_{it}\ (1-\tau)$$

where τ denotes a subsidy per unit of labor received from the government. This subsidy is introduced so that under flexible prices the monopolistic firm would produce the competitive level of output. In this way, the only distortion remaining in the model is the one associated with sluggish price adjustment. While this assumption, which is customary in the neo-Keynesian literature, greatly facilitates the characterization of optimal monetary policy, it is not crucial in deriving the main results of this section.

The firm must satisfy demand at posted prices. Formally, this requirement gives rise to the restriction

$$z_t h_{it} \geq C_{it},$$

where, as derived earlier, c_{it} is given by $c_{it} = \left(\frac{Q_{it}}{Q_t}\right)^{-\eta} \frac{a_t}{x_{it}}$. Let MC_{it} denote the Lagrange multiplier on the above constraint. Then, the optimality condition of the firm's problem with respect to labor is given by

$$(1 - \tau)\ W_t = MC_{it} z_t.$$

It is clear from this first-order condition that MC_{it} must be identical across firms. We therefore drop the subscript i from this variable.

Consider now the price setting problem of the monopolistically competitive firm. For the purpose of determining the optimal inflation target, it is crucial to be precise in regard to what prices are assumed to be costly to adjust. We distinguish two cases. In one case we assume that nonquality-adjusted prices, P_{it}, are sticky. In the second case, we assume that quality-adjusted (or hedonic) prices, Q_{it}, are sticky. Using the example of the personal computer again, the case of stickiness in nonquality-adjusted prices would correspond to a situation in which the price of the personal computer is costly to adjust. The case of stickiness in quality-adjusted prices results when the price of a computer per unit of quality is sticky, where in our example quality would be measured by an index capturing attributes such as memory, processing speed, video capabilities, and so forth. We consider first the case in which stickiness occurs at the level of nonquality-adjusted prices.

10.2 Stickiness in nonquality-adjusted prices

Suppose that with probability α firm $i \in [0,1]$ cannot reoptimize its price, P_{it}, in a given period. Consider the price-setting problem of a firm that has the chance to reoptimize its price in period t. Let $\widetilde{P}_{it}$ be the price chosen by such firm. The portion of the Lagrangian associated with the firm's optimization problem that is relevant for the purpose of determining $\widetilde{P}_{it}$ is given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j}\, \alpha^j \left[\left(\frac{\eta - 1}{\eta}\right) \widetilde{P}_{it} - MC_{t+j}\right] \left(\frac{\widetilde{P}_{it}}{x_{it+j} Q_{t+j}}\right)^{-\eta} \frac{a_{t+j}}{x_{it+j}} = 0.$$

The first-order condition with respect to $\widetilde{P}_{it}$ is given by

$$E_t \sum_{j=0}^{\infty} r_{t,t+j}\, \alpha^j \left[\left(\frac{\eta-1}{\eta}\right) \widetilde{P}_{it} \;-\; MC_{t+j}\right] \left(\frac{\widetilde{P}_{it}}{P_{t+j}}\right)^{-\eta} c_{t+j} \;=\; 0.$$

Although we believe that the case of greatest empirical interest is one in which quality varies across goods, maintaining such an assumption complicates the aggregation of the model, as it adds another source of heterogeneity in addition to the familiar price dispersion stemming from Calvo-Yun staggering. Consequently, to facilitate aggregation, we assume that all goods are of the same quality; that is, we assume that $x_{it} = x_t$ for all i. We further simplify the exposition by assuming that x_t grows at the constant rate $\kappa > 0$, that is,

$$x_t = (1 \;+\; \kappa)x_{t-1}.$$

In this case, the above first-order condition simplifies to

$$E_t \sum_{j=0}^{\infty} r_{t,t+j}\, \alpha^j \left[\left(\frac{\eta-1}{\eta}\right) \widetilde{P}_{it} \;-\; MC_{t+j}\right] \left(\frac{\widetilde{P}_{it}}{P_{t+j}}\right)^{-\eta} c_{t+j} \;=\; 0.$$

where

$$C_t \;\equiv\; \left[\int_0^1 C_{it}^{1-1/\eta} di\right]^{1/(1-1/\eta)}$$

and

$$P_t \;\equiv\; \left[\int P_{it}^{1-\eta} di\right]^{1/(1-\eta)}.$$

It is clear from these expressions that all firms that have the chance to reoptimize their price in a given period will choose the same price. We therefore drop the subscript i from the variable $\widetilde{P}_{it}$. We also note that the definitions of P_t and c_t imply that $P_t\, C_t = \int_0^1 P_{it} C_{it} di$. Thus P_t can be interpreted as the CPI unadjusted for quality improvements.

The aggregate price level P_t is related to the reoptimized price $\widetilde{P}_t$ by the following familiar expression in the Calvo-Yun framework:

$$P_t^{1-\eta} = \alpha P_{t-1}^{1-\eta} \;+\; (1-\alpha)\widetilde{P}_t^{1-\eta}.$$

Market clearing for good i requires that

$$z_t h_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} C_t.$$

Integrating over $i \in [0,1]$ yields

$$z_t h_t = C_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di,$$

Where $h_t \equiv \int_0^1 h_{it} di$. Letting $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$, we can write the aggregate resource constraint as

$$z_t h_t = s_t c_t,$$

where, as shown earlier in Section 6, s_t measures the degree of price dispersion in the economy and obeys the law of motion

$$s_t = (1-\alpha)\widetilde{P}_t^{-\eta} + \alpha\pi_t^{\eta} s_{t-1},$$

where $\widetilde{P}_t \equiv \widetilde{P}_t / P_t$ denotes the relative price of goods whose price was reoptimized in period t, and $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of inflation in period t not adjusted for quality improvements.

A competitive equilibrium is a set of processes c_t, h_t, mc_t, s_t, and $\widetilde{p}$ satisfying

$$-\frac{U_2(x_t c_t,\ h_t)}{U_1(x_t c_t,\ h_t)} = \frac{\mathrm{mc}_t z_t x_t}{1-\tau},$$
$$z_t h_t = s_t c_t,$$
$$s_t = (1-\alpha)\widetilde{P}_t^{-\eta} + \alpha\pi_t^{\eta} s_{t-1},$$
$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\widetilde{p}_t^{1-\eta},$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_1(x_s c_s\ h_s)}{U_1(x_t c_t\ h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right)^{-\eta} x_s c_s \left[\mathrm{mc}_s - \left(\frac{\eta-1}{\eta}\right)\widetilde{p}_t \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right)\right] = 0,$$

given exogenous processes z_t and x_t and a policy regime π_t. The variable $mc_t = MC_t/P_t$ denotes the marginal cost of production in terms of the composite good c_t.

We now establish that when nonquality-adjusted prices are sticky, the Ramsey optimal monetary policy calls for not incorporating the quality bias into the inflation target. That is, the optimal monetary policy consists in constant nonquality-adjusted prices. To this end, as in previous sections, we assume that $s_{-1} = 1$, so that there is no inherited price dispersion in period 0. Set $\pi_t = 1$ for all t and $1 - \tau = (\eta - 1)/\eta$. By the same arguments given in Section 6.2, the preceding equilibrium conditions become identical to those associated with the problem of maximizing $E_0 \sum_{t=0}^{\infty} \beta^t U(x_t c_t,\ h_t)$, subject to $z_t h_t = c_t$. We have therefore demonstrated that setting π_t equal to unity is not only Ramsey optimal but also Pareto efficient.

Importantly, π_t is the rate of inflation that results from measuring prices without adjusting for quality improvement. The inflation rate that takes into account

Table 4 The Optimal Rate of Inflation Under Quality Bias

	Statistical agency	
	Corrects quality bias	
Stickiness in	No	Yes
nonquality-adjusted prices	0	$-\kappa$
Quality-adjusted (or hedonic) prices	κ	0

Note: The parameter $\kappa > 0$ denotes the rate of quality improvement.

improvements in the quality of goods is given by Q_t/Q_{t-1}, which equals $\pi_t/(1+\kappa)$ and is less than π_t by our maintained assumption that quality improves over time at the rate $\kappa > 0$. Therefore, although there is a quality bias in the measurement of inflation, given by the rate of quality improvement κ, the central bank should not target a positive rate of inflation.

This result runs contrary to the usual argument that in the presence of a quality bias in the aggregate price level, the central bank should adjust its inflation target upwards by the magnitude of the quality bias. For instance, suppose that, in line with the findings of the Boskin Commission, the quality bias in the rate of inflation was 0.6% (or $\kappa = 0.006$). Then, the conventional wisdom would suggest that the central bank of the economy analyzed in this section target a rate of inflation of about 0.6%. We have shown, however, that such policy would be suboptimal. Rather, optimal policy calls for a zero inflation target. The key to understanding this result is to identify exactly which prices are sticky. For optimal policy aims at keeping the price of goods that are sticky constant over time to avoid inefficient price dispersion. Here we are assuming that stickiness originates in nonqualityadjusted prices. Therefore, optimal policy consists in keeping these prices constant over time. At the same time, because quality-adjusted (or hedonic) prices are flexible, the monetary authority can let them decline at the rate κ without creating distortions.

Suppose now that the statistical agency responsible for constructing the CPI decided to correct the index to reflect quality improvements. For example, in response to the publication of the Boskin Commission report, the U.S. Bureau of Labor Statistics reinforced its use of hedonic prices in the construction of the CPI. In the ideal case in which all of the quality bias is eliminated from the CPI, the statistical agency would publish data on Q_t rather than on P_t. How should the central bank adjust its inflation target in response to this methodological advancement? The goal of the central bank continues to be the complete stabilization of the nonquality-adjusted price, P_t, for this is the price that suffers from stickiness. To achieve this goal, the published price index, Q_t, would have to be falling at the rate of quality improvement, κ. This means that the central bank would have to target deflation at the rate κ.

To summarize, when nonquality-adjusted prices are sticky, the optimal inflation target of the central bank is either zero (when the statistical agency does not correct the price index for quality improvements) or negative at the rate of quality improvement (when the statistical agency does correct the price index for quality improvements; see Table 4).

10.3 Stickiness in quality-adjusted prices

Assume now that quality-adjusted (or hedonic) prices, Q_{it}, are costly to adjust. Consider the price-setting problem of a firm, i say, that has the chance to reoptimize Q_{it} in period t. Let $\widetilde{Q}_{it}$ be the quality-adjusted price chosen by such firm. The portion of the Lagrangian associated with the firm's profit maximization problem relevant for the purpose of determining the optimal level of $\widetilde{Q}_{it}$ is given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j}\, \alpha^j \left[\widetilde{Q}_{it} x_{t+j} - MC_{t+j}\right] \left(\frac{\widetilde{Q}_{it}}{Q_{t+j}}\right)^{-\eta} c_{t+j}.$$

The first-order condition with respect to $\widetilde{Q}_{it}$ is given by

$$E_t \sum_{j=0}^{\infty} r_{t,t+j}\, \alpha^j \left[\left(\frac{\eta-1}{\eta}\right) \widetilde{Q}_{it} x_{t+j} - MC_{t+j}\right] \left(\frac{\widetilde{Q}_{it}}{Q_{t+j}}\right)^{-\eta} c_{t+j} = 0.$$

A competitive equilibrium in the economy with stickiness in quality-adjusted prices is a set of processes c_t, h_t, mc_t, s_t, and $\widetilde{P}_t$ that satisfy

$$-\frac{U_2(x_t c_t, h_t)}{U_1(x_t c_t, h_t)} = \frac{\mathrm{mc}_t z_t x_t}{1-\tau}$$

$$z_t h_t = s_t c_t$$

$$s_t = (1-\alpha)(\widetilde{p}_t)^{-\eta} + \alpha\left(\pi_t \frac{x_{t-1}}{x_t}\right)^{\eta} s_{t-1},$$

$$1 = \alpha \pi_t^{\eta-1}\left(\frac{x_t}{x_{t-1}}\right)^{1-\eta} + (1-\alpha)(\widetilde{p}_t)^{1-\eta},$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_1(x_s c_s, h_s)}{U_1(x_t c_t, h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right)^{-\eta} x_s c_s \left[\mathrm{mc}_s - \left(\frac{\eta-1}{\eta}\right) \widetilde{p}_t \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right) \frac{x_s}{x_t}\right] = 0,$$

given exogenous processes z_t and x_t and a policy regime π_t.

We wish to demonstrate that when quality-adjusted prices are sticky, the optimal rate of inflation is positive and equal to the rate of quality improvement, κ.

Again assume no initial dispersion of relative prices by setting $s_{-1} = 1$. Then, setting $\pi_t = x_t/x_{t-1}$, we have that in the competitive equilibrium $\widetilde{P}_t = 1$ and $s_t = 1$ for all t. Assuming further that the fiscal authority sets $1 - \tau = (\eta - 1)/\eta$, we have that the set of competitive equilibrium conditions becomes identical to the set of optimality conditions associated with the social planner's problem of maximizing

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t c_t, h_t), \text{ subject to } z_t h_t = c_t$$

We have therefore proven that when quality-adjusted prices are sticky, a positive inflation target equal to the rate of quality improvement ($\pi_t = 1 + \kappa$) is Ramsey optimal and Pareto efficient. In this case, the optimal adjustment in the inflation target conforms to the conventional wisdom, according to which the quality bias in inflation measurement justifies an upward correction of the inflation target equal in size to the bias itself. The intuition behind this result is that in order to avoid relative price dispersion, the monetary authority must engineer a policy where firms have no incentives to change prices that are sticky. In the case considered here the prices that are sticky happen to be the quality-adjusted prices. At the same time, nonquality- adjusted prices are fully flexible and therefore under the optimal policy they are allowed to grow at the rate κ without creating inefficiencies.

Finally, suppose that the statistical agency in charge of preparing the CPI decided to correct the quality bias built into the price index. In this case, the central bank should revise its inflation target downward to zero in order to accomplish its goal of price stability in (sticky) quality-adjusted prices. Table 4 summarizes the results of this section.

We interpret the results derived in this section as suggesting that if the case of greatest empirical relevance is one in which nonquality-adjusted prices (the price of the personal computer in the example we have been using throughout) is sticky, then the conventional wisdom that quality bias justifies an upward adjustment in the inflation target is misplaced. Applying this conclusion to the case of the United States, it would imply that no fraction of the 2% inflation target implicit in Fed policy is justifiable on the basis of the quality bias in the U.S. CPI. Moreover, the corrective actions taken by the Bureau of Labor Statistics in response to the findings of the Boskin commission, including new hedonic indexes for television sets and personal computers as well as an improved treatment-based methodology for measuring medical care prices, would actually justify setting negative inflation targets. If, on the other hand, the more empirically relevant case is the one in which hedonic prices are sticky, then the conventional view that the optimal inflation target should be adjusted upward by the size of the quality bias is indeed consistent with the predictions of our model. The central empirical question raised by the theoretical analysis presented in this section is therefore whether regular or hedonic prices are more sticky. The existing empirical literature on nominal price rigidities has yet to address this matter.

11. CONCLUSION

This chapter addressed the question whether observed inflation targets around the world, ranging from 2% in developed countries to 3.5% in developing countries, can be justified on welfare-theoretic grounds. The two leading sources of monetary non-neutrality in modern models of the monetary transmission mechanism — the demand for money and sluggish price adjustment — jointly predict optimal inflation targets of at most 0% per year.

Additional reasons frequently put forward in explaining the desirability of inflation targets of the magnitude observed in the real world — including incomplete taxation, the zero lower bound on nominal interest rates, downward rigidity in nominal wages, and a quality bias in measured inflation — are shown to deliver optimal rates of inflation insignificantly above zero.

Our analysis left out three potentially relevant theoretical considerations bearing on the optimal rate of inflation. One is heterogeneity in income across economic agents. To the extent that the income elasticity of money demand is less than unity, lower income agents will hold a larger fraction of their income in money than high income agents. As a result, under these circumstances the inflation rate acts as a regressive tax. This channel, therefore, is likely to put downward pressure on the optimal rate of inflation, insofar as the objective function of the policymaker is egalitarian.

A second theoretical omission in our analysis concerns heterogeneity in consumption growth rates across regions in a monetary union. To the extent that the central bank of the monetary union is concerned with avoiding deflation, possibly because of downward nominal rigidities, it will engineer a monetary policy consistent with price stability in the fastest growing region. This policy implies that all other regions of the union will experience inflation until differentials in consumption growth rates have disappeared. To our knowledge, this argument has not yet been evaluated in the context of an estimated dynamic model of a monetary union. But perhaps more important, this channel would not be useful to explain why small, relatively homogeneous countries, such as New Zealand, Sweden, or Switzerland, have chosen inflation targets similar in magnitude to those observed in larger, less homogeneous, currency areas such as the United States or the Euro Area. Here one might object that the small countries are simply following the leadership of the large countries. However, the pioneers in setting inflation targets of 2% were indeed small countries like New Zealand, Canada, and Sweden.

A third theoretical channel left out from our investigation is time inconsistency on the part of the monetary policy authority. Throughout our analysis, we assume that the policymaker has access to a commitment technology that ensures that all policy announcements are honored. Our decision to restrict attention to the commitment case is twofold: First, the commitment case provides the optimum optimarum inflation target, which serves as an important benchmark. Second, it is our belief that political and economic institutions in industrial countries have reached a level of development at which central bankers find it in their own interest to honor past promises. In other words, we believe that it is realistic to model central bankers as having access to some commitment technology, or, as Blinder (1999) observed, "enlightened discretion is the rule.'

APPENDIX

1 Derivation of the primal form of the model with a demand for money and fiscal policy of Section 3

We first show that plans $\{c_t, h_t, v_t\}$ satisfying the equilibrium conditions (2), (4) holding with equality, (5), (7), (8), (11), and (13)–(15) also satisfy (14), (16), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$. Let $\gamma(u_t) \equiv 1 + s(u_t) + u_t s'(u_t)$. Note that Eqs. (5), (11), and our maintained assumptions regarding $s(v)$ together imply that $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$. Let $W_{t+1} = R_t B_t + M_t$. Use this expression to eliminate B_t from (15) and multiply by $q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$ to obtain

$$q_t M_t\left(1 - R_t^{-1}\right) + q_{t+1} W_{t+1} - q_t W_t = q_t\left[P_t g_t - \tau_t^h P_t w_t h_t\right].$$

Sum for $t = 0$ to $t = T$ to obtain

$$\sum_{t=0}^{T}\left[q_t M_t\left(1 - R_t^{-1}\right) - q_t\left(P_t g_t - \tau_t^h P_t w_t h_t\right)\right] = q_{T+1} W_{T+1} + W_0.$$

In writing this expression, we define $q_0 = 1$.

Take limits for $T \to \infty$. By Eq. (4) holding with equality the limit of the right hand side is well defined and equal to W_0. Thus, the limit of the left-hand side exists. This yields:

$$\sum_{t=0}^{\infty}\left[q_t M_t\left(1 - R_t^{-1}\right) - q_t\left(P_t g_t - \tau_t^h P_t w_t h_t\right)\right] = W_0$$

By Eq. (7) we have that $P_t q_t = \beta^t U_c(c_t,\ h_t)/\gamma(u_t) P_0 / U_c(c_0, h_0)\gamma(v_0)$. Use this expression to eliminate $P_t q_t$ from the above equation. Also, use (2) to eliminate M_t/P_t to obtain

$$\sum_{t=0}^{\infty}\beta^t \frac{U_c(c_t, h_t)}{\gamma(v_t)}\left[\frac{c_t}{v_t}\left(1 - R_t^{-1}\right) - \left(g_t - \tau_t^h w_t h_t\right)\right] - \frac{W_0}{P_0}\frac{U_c(c_0, h_0)}{\gamma(v_0)}$$

Solve Eq. (13) for τ_t^h and Eq. (8) for w_t to obtain $\tau_t^h w_t h_t = F'(h_t) h_t + \gamma(v_t)/U_c(c_t, h_t) U_h(c_t, h_t) h_t$. Use this expression to eliminate $\tau_t^h w_t h_t$ from the above equation. Also use Eq. (5) to replace $\left(1 - R_t^{-1}\right)/v_t$ with $v_t s'(v_t)$, and replace g_t with Eq. (14). This yields

$$\sum_{t=0}^{\infty}\beta^t\left[U_c(c_t,h_t)c_t+U_h(c_t,h_t)h_t+\frac{U_c(c_t,h_t)}{\gamma(v_t)}[F'(h_t)h_t-F(h_t)]\right]=\frac{W_0}{P_0}\frac{U_c(c_0,h_0)}{\gamma(v_0)}$$

Finally, use $W_0 = R_{-1}\, B_{-1} + M_{-1}$ to obtain

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^t\left[U_c(c_t,h_t)c_t+U_h(c_t,h_t)h_t+[F'(h_t)h_t-F(h_t)]\frac{U_c(c_t,h_t)}{\gamma(v_t)}\right]\\&=\left(\frac{R_{-1}B_{-1}+M_{-1}}{P_0}\right)\left(\frac{U_c(c_0,h_0)}{\gamma(v_0)}\right)\end{aligned}$$

which is Eq. (16).

Now we show that plans $\{c_t, h_t, v_t\}$ that satisfy $v_t \geq \underline{v}, v_t^2 s'(v_t) < 1$, Eqs. (14), and (16) also satisfy Eqs. (2), (4) holding with equality, (5), (7), (8), (11), and (13)–(15) at all dates.

Given a plan $\{c_t, h_t, v_t\}$ proceed as follows. Use Eq. (5) to construct R_t as $1/\left[1-v_t^2 s'(v_t)\right]$. Note that under the maintained assumptions on $s(v)$, the constraints $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$ ensure that $R_t \geq 1$. Let w_t be given by Eq. (8) and τ_t^h by Eq. (13). To construct plans for M_t, P_{t+1}, and B_t, for $t \geq 0$, use the following iterative procedure: (a) Set $t = 0$, (b) use Eq. (2) to construct M_t (one can do this for $t = 0$ because P_0 is given), (c) set B_t so as to satisfy Eq. (15), (d) set P_{t+1} to satisfy Eq. (7), (e) increase t by 1 and repeat steps (b) through (e). This procedure yields plans for P_t and thus for the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. It remains to be shown that Eq.(4) holds with equality. Sum (15) for $t = 0$ to $t = T$, which as shown above, yields:

$$\begin{aligned}&\sum_{t=0}^{T}\beta^t\left[U_c(c_t,h_t)c_t+U_h(c_t,h_t)h_t+[F'(h_t)h_t-F(h_t)]\frac{U_c(c_t,h_t)}{\gamma(v_t)}\right]=\\&\left(\frac{-q_{T+1}W_{T+1}+R_{-1}B_{-1}+M_{-1}}{P_0}\right)\frac{U_c(c_0,h_0)}{\gamma(v_0)}\end{aligned}$$

By Eq. (16) the limit of the left-hand side of this expression as $T \to \infty$ exists and is equal to $\frac{R_{-1}B_{-1}+M_{-1}}{P_0}\frac{U_c(c_0,h_0)}{\gamma(v_0)}$. Thus the limit of the right-hand side also exists and we have

$$\lim_{T\to\infty} q_{T+1}W_{T+1}=0$$

which is Eq. (4). This completes the proof.

2 Derivation of the primal form in the model with a foreign demand for domestic currency of Section 5

We first show that plans $\{c_t, h_t, v_t\}$ satisfying the equilibrium conditions (2), (4) holding with equality, (5), (7), (8), (11), (13), and (25)–(28) also satisfy (29), (30), (31), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$. Note that, as in the case without a foreign demand for currency, Eqs. (5), (11), and our maintained assumptions regarding $s(v)$ together imply that $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$.

Let $W_{t+1} = R_t B_t + M_t + M_t^f$. Use this expression to eliminate B_t from Eq. (27) and multiply by $q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$ to obtain

$$q_t\left(M_t + M_t^f\right)\left(1 - R_t^{-1}\right) + q_{t+1} W_{t+1} - q_t W_t = q_t\left[P_t g_t - \tau_t^h P_t F(h_t)\right].$$

Sum for $t = 0$ to $t = T$ to obtain

$$\sum_{t=0}^{T}\left[q_t\left(M_t + M_t^f\right)\left(1 - R_t^{-1}\right) - q_t\left(P_t g_t - \tau_t^h P_t F(h_t)\right)\right] = -q_{T+1} W_{T+1} + W_0.$$

In writing this expression, we define $q_0 = 1$. Solve Eq. (13) for τ_t^h and Eq. (8) for w_t and use $F(h) = h$ to obtain $\tau_t^h F(h_t) = h_t + \frac{U_h(c_t,h_t)}{U_c(c_t,h_t)}\gamma(v_t)h_t$. Use this expression to eliminate $\tau_t^h F(h_t)$ from the above equation, which yields

$$\sum_{t=0}^{T}\left\{q_t\left(M_t + M_t^f\right)\left(1 - R_t^{-1}\right) - q_t P_t\left[g_t - \left[h_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t\right]\right]\right\}$$
$$= -q_{T+1} W_{T+1} + W_0.$$

Use the feasibility constraint (28) to replace $h_t - g_t$ with $[1 + s(v_t)]c_t - \frac{M_t^f - M_{t-1}^f}{P_t}$.

$$\sum_{t=0}^{T} q_t P_t\left\{\frac{M_t + M_t^f}{P_t}\left(1 - R_t^{-1}\right) + [1 + s(v_t)]c_t - \frac{M_t^f - M_{t-1}^f}{P_t} + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t\right\}$$
$$= -q_{T+1} W_{T+1} + W_0.$$

Use Eqs. (2) and (5) to replace $\frac{M_t}{P_t}\left(1 - R_t^{-1}\right)$ with $v_t s'(v_t)c_t$

$$\sum_{t=0}^{T} q_t P_t\left\{v_t s'(v_t)c_t - \frac{M_t^f}{P_t R_t} + [1 + s(v_t)]c_t + \frac{M_{t-1}^f}{P_t} + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t\right\}$$
$$= -q_{T+1} w_{T+1} + W_0.$$

Collect terms in c_t and replace $1 + s(v_t) + v_t s'(v_t)$ with $\gamma(v_t)$ and rearrange.

Noting that by definition $q_t/R_t = q_{t+1}$ write the above expression as

$$\sum_{t=0}^{T} q_t P_t\left\{\gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t - \frac{M_t^f}{P_t R_t} + \frac{M_{t-1}^f}{P_t}\right\} = -q_{T+1} W_{T+1} + W_0.$$

Evaluate the second sum on the left-hand side and recall that by definition $q_0 = 1$ to obtain

$$\sum_{t=0}^{T} q_t P_t\left\{\gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t\right\} + M_{-1}^f - M_T^f q_{T+1}$$
$$= -q_{T+1} W_{T+1} + W_0.$$

Using the definition of W_t we can write the above expression as:

$$\sum_{t=0}^{T} q_t\, P_t \left\{ \gamma(v_t)\, c_t + \frac{U_h\,(c_t,\; h_t)}{U_c\,(c_t,\; h_t)}\, \gamma\,(v_t)\, h_t \right\} = -\, q_{T+1}\,(R_T B_T + M_T) + R_{-1} B_{-1} + M_{-1}. \tag{65}$$

Take limits for $T \to \infty$. Then by Eq. (4) holding with equality the limit of the right-hand side is well defined and equal to $R_{-1}B_{-1} + M_{-1}$. Thus, the limit of the left-hand side exists. This yields:

$$\sum_{t=0}^{\infty} q_t P_t \left\{ \gamma\,(v_t)\, c_t + \frac{U_h\,(c_t,\; h_t)}{U_h\,(c_t,\; h_t)}\, \gamma\,(v_t)\, h_t \right\} = R_{-1}B_{-1} + M_{-1}.$$

By Eq. (7) we have that $P_t q_t = \beta^t U_c(c_t,\, h_t)/\gamma(v_t) P_0 / U_c(c_0,\, h_0)\gamma(v_0)$. Use this expression to eliminate $P_t q_t$ from the above equation to obtain

$$\sum_{t=0}^{\infty} \beta^t [U_c(c_t, h_t)c_t + U_h(c_t, h_t)h_t] = \left(\frac{U_c(c_0, h_0)}{\gamma(v_0)} \right) \left(\frac{R_{-1}B_{-1} + M_{-1}}{P_0} \right),$$

which is Eq. (31).

We next show that the competitive equilibrium conditions imply Eqs. (29) and (30). Equation (29) follows directly from Eq. (26) and the definition of $\chi(v_t)$ given in Eq. (32). For $t > 0$, use Eq. (26) to eliminate M_t^f and M_{t-1}^f from Eq. (28) to obtain:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{y_t^f}{v_t^f} - \frac{y_{t-1}^f}{v_{t-1}^f}\,\frac{1}{\pi_t}.$$

Now use Eq.(7) to eliminate π_t. This yields:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{y_t^f}{\chi(v_t)} - \frac{y_{t-1}^f}{\chi(v_{t-1})}\,\frac{U_c\,(c_{t-1}, h_{t-1})}{R_{t-1}\,\gamma(v_{t-1})}\,\frac{\gamma(v_t)}{\beta U_c(c_t,\; h_t)},$$

Using Eq. (5) to replace R_{t-1} yields Eq. (30). This completes the proof that the competitive equilibrium conditions imply the primal form conditions.

We now show that plans $\{c_t,\; h_t,\; v_t\}$ satisfying Eqs. (29), (30), (31), $v_t \geq \underline{v}$, and $v_t^2 s'\,(v_t) < 1$ also satisfy the equilibrium conditions (2), (4) holding with equality, (5), (7), (8), (11), (13), and (25)–(28). Given a plan $\{c_t,\; h_t,\; v_t\}$ proceed as follows. Use Eq. (5) to construct R_t and Eq. (25) to construct u_t^f. Note that under the maintained assumptions on $s(v)$, the constraints $v_t \geq \underline{v}$ and $v_t^2 s'\,(v_t) < 1$ ensure that $R_t \geq 1$. Let w_t be given by Eq. (8) and τ_t^h by Eq. (13).

To construct plans for M_t, M_t^f, P_{t+1}, and B_t, for $t \geq 0$, use the following iterative procedure: (a) set $t = 0$, (b) use Eq. (2) to construct M_t and Eq. (26) to construct M_t^f

(recall that P_0 is given), (c) set B_t so as to satisfy Eq. (27); (d) set P_{t+1} to satisfy Eq. (7), (e) increase t by 1 and repeat steps (b) through (e). Next we want to show that Eq. (28) holds. First we want to show that it holds for $t=0$. Combining Eqs. (26) and (32) with Eq. (29) it is obvious that Eq. (28) holds for $t=0$. To show that it also holds for $t>0$, combine Eqs. (26), (32), and (30) to obtain:

$$\begin{aligned}[1+s(v_t)]\,c_t &= g_t \\ &= F(h_t) + \frac{M_t^f}{P_t} \\ &\quad - \frac{M_{t-1}^f}{P_{t-1}}\left(1 - v_{t-1}^2\, s'(v_{t-1})\right)\frac{U_c(c_{t-1},\, h_{t-1})}{\gamma(v_{t-1})}\,\frac{\gamma(v_t)}{\beta U_c\, 9c_t,\, h_t)},\end{aligned}$$

Using Eq. (5) this expression can be written as:

$$[1+s(v_t)]\,c_t = g_t = F(h_t) + \frac{M_t^f}{P_t} - \frac{M_{t-1}^f}{P_{t-1}}(1/R_{t-1})\frac{U_c(c_{t-1},\, h_{t-1})}{\gamma(v_{t-1})}\,\frac{\gamma(v_t)}{\beta U_c 9c_t,\, h_t)},$$

Finally, combining this expression with Eq. (7) yields Eq. (28).

It remains to be shown that Eq. (4) holds with equality. Follow the preceding steps to arrive at Eq. (65). Notice that these steps make use only of equilibrium conditions that we have already shown are implied by the primal form. Now use Eq. (7) (which we have already shown to hold) to replace $P_t q_t$ with $\beta^t U_c(c_t, h_t)/\gamma(v_t)P_0/U_c(c_0, h_0)\gamma(v_0)$ to obtain

$$\begin{aligned}\sum_{t=0}^{T}\beta^t\,[U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t] = -\,q_{T+1}\,(R_T\,B_T + M_T)\left(\frac{U_c(c_0,h_0)}{P_0\gamma(v_0)}\right) \\ + \left(\frac{U_c(c_0,\, h_0)}{\gamma(v_0)}\right)\left(\frac{R_{-1}B_{-1} + M_{-1}}{P_0}\right).\end{aligned}$$

Taking limit for $T\to\infty$, recalling the definition of q_t, and using Eq. (31) yields Eq. (4) holding with equality. This completes the proof.

REFERENCES

Adam, K., Billi, R.M., 2006. Optimal monetary policy under commitment with a zero bound on nominal interest rates. J. Money Credit Bank. 38, 1877–1905.

Akerlof, G.A., Dickens, W.T., Perry, G.L., 1996. The macroeconomics of low inflation. Brookings Pap. Econ. Act. 1–76.

Altig, D., Christiano, L.J., Eichenbaum, M., Lindé, J., 2005. Firm-specific capital, nominal rigidities, and the business cycle. NBER WP 11034.

Basu, S., Fernald, J.G., 1997. Returns to scale in U.S. production: Estimates and implications. J. Polit. Econ. 105, 249–283.

Benigno, P., Woodford, M., 2005. Inflation stabilization and welfare: The case of a distorted steady state. J. Eur. Econ. Assoc. 3, 1185–1236.

Bils, M., Klenow, P., 2004. Some evidence on the importance of sticky prices. J. Polit. Econ. 112, 947–985.

Blinder, A., 1999. Central banking in theory and practice. MIT Press, Cambridge, MA.

Calvo, G., 1983. Staggered prices in a utility-maximizing framework. J. Monet. Econ. 12, 383–398.

Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. J. Polit. Econ. 107, 205–251.

Card, D., Hyslop, D., 1997. Does inflation "grease the wheels of the labor market"? In: Romer, C., Romer, D. (Eds.), Reducing inflation: Motivation and strategy. University of Chicago Press, Chicago, pp. 71–122.

Chari, V.V., Christiano, L., Kehoe, P., 1991. Optimal fiscal and monetary policy: Some recent results. J. Money Credit Bank. 23, 519–539.

Chari, V.V., Christiano, L., Kehoe, P., 1996. Optimality of the Friedman rule in economics with distorting taxes. J. Monetary Econ. 37, 203–223.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. J. Polit. Econ. 113, 1–45.

Cogley, T., Sbordone, A.M., 2008. Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. Am. Econ. Rev. 98, 2101–2126.

Correia, I., Teles, P., 1996. Is the Friedman rule optimal when money is an intermediate good? J. Monet. Econ. 38, 223–244.

Correia, I., Teles, P., 1999. The optimal inflation tax. Rev. Econ. Dyn. 2, 325–346.

Correia, I., Nicolini, J.P., Teles, P., 2008. Optimal fiscal and monetary policy: Equivalence results. J. Polit. Econ. 168, 141–170.

Del Negro, M., Schorfheide, F., Smets, F., Wouters, R., 2004. On the fit and forecasting performance of New-Keynesian models. Manuscript.

Feenstra, R.C., 1986. Functional equivalence between liquidity costs and the utility of money. J. Monet. Econ. 17, 271–291.

Goodfriend, M., King, R.G., 1997. The new neoclassical synthesis and the role of monetary policy. In: Bernanke, B., Rotemberg, J.J. (Eds.), NBER macroeconomics annual 1997. MIT Press, Cambridge MA, pp. 231–283.

Guidotti, P.E., Végh, C.A., 1993. The optimal inflation tax when money reduces transactions costs: A reconsideration. J. Monet. Econ. 31, 189–205.

Khan, A., King, R.G., Wolman, A., 2003. Optimal monetary policy. Rev. Econ. Stud. 70, 825–860.

Kim, J., Ruge-Murcia, F.J., 2009. How much inflation is necessary to grease the wheels? J. Monetary Econ. 56, 365–377.

Kimbrough, K.P., 1986. The optimum quantity of money rule in the theory of public finance. J. Monet. Econ. 18, 277–284.

King, R.G., Wolman, A.L., 1999. What should the monetary authority do when prices are sticky? In: Taylor, J.B. (Ed.), Monetary policy rules. University of Chicago Press, Chicago, pp. 349–398.

Klenow, P., Kryvtsov, O., 2005. State-dependent or time-dependent pricing: Does it matter for recent U.S. inflation?. Stanford University, Mimeo.

Levin, A., Onatski, A., Williams, J., Williams, N., 2006. Monetary policy under uncertainty in micro-founded macroeconometric models. In: Mark, G., Rogoff, K. (Eds.), NBER macroeconomics annual, 2005, 20, MIT Press, Cambridge.

Lucas, R.E., 1982. Interest rates and currency prices in a two-country world. J. Monet. Econ. 10, 335–360.

McLaughlin, K.J., 1994. Rigid Wages? J. Monet. Econ. 34, 383–414.

Nakamura, E., Steinsson, J., 2007. Five facts about prices: A reevaluation of menu cost models. Harvard University, Mimeo.

Nicolini, J.P., 1998. Tax evasion and the optimal inflation tax. J. Dev. Econ. 55, 215–232.

Olivera, J.H.G., 1960. La teoría no monetaria de la inflatión. Trimest. Econ. 27, 616–628.

Olivera, J.H.G., 1964. On structural inflation and Latin-American "structuralism" Oxf. Econ. Pap. 16, 321–332.

Phelps, E.S., 1973. Inflation in the theory of public finance. The Swedish Journal of Economics 75, 67–82.
Porter, R.D., Judson, R.A., 1996. The location of U.S. currency: How much is abroad? Federal Reserve Bulletin 883–903.
Ravn, M., 2005. Labor market matching, labor market participation and aggregate business cycles: Theory and structural evidence for the United States. Manuscript European University Institute.
Reifschneider, D., Williams, J.C., 2000. Three lessons for monetary policy in a low-inflation era. J. Money Credit Bank. 32, 936–966.
Rotemberg, J.J., 1982. Sticky prices in the United States. J. Polit. Econ. 90, 1187–1211.
Schmitt-Grohé, S., Uribe, M., 2004a. Optimal fiscal and monetary policy under sticky prices. J. Econ. Theory 114, 198–230.
Schmitt-Grohé, S., Uribe, M., 2004b. Optimal fiscal and monetary policy under imperfect competition. J. Macroecon. 26, 183–209.
Schmitt-Grohé, S., Uribe, M., 2006. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In: Gertler, M., Rogoff, K. (Eds.), NBER macroeconomics annual 2005. MIT Press, Cambridge and London, pp. 383–425.
Schmitt-Grohé, S., Uribe, M., 2007a. Optimal simple and implementable monetary and fiscal rules. J. Monet. Econ. 54, 1702–1725.
Schmitt-Grohé, S., Uribe, M., 2007b. Optimal inflation stabilization in a medium-scale macroeconomic model. In: Schmidt-Hebbel, K., Mishkin, R. (Eds.), Monetary Policy Under Inflation Targeting. Central Bank of Chile, Santiago, Chile, pp. 125–186.
Schmitt-Grohé, S., Uribe, M., 2009a. Foreign demand for domestic currency and the optimal rate of inflation. NBER working paper 15494.
Schmitt-Grohé, S., Uribe, M., 2009b. On quality bias and inflation targets. NBER working paper 15505.
Sidrauski, M., 1967. Rational choice and patterns of growth in a monetary economy. American Economic Review, Papers and Proceedings 57, 534–544.
Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro Area. J. Eur. Econ. Assoc. 1, 1123–1175.
Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach. Am. Econ. Rev. 97, 586–606.
Summers, L., 1991. How should long-term monetary policy be determined? J. Money Credit Bank. 23, 625–631.
Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie Rochester Conference Series on Public Policy 39, 195–214.
Tobin, J., 1972. Inflation and unemployment. Am. Econ. Rev. 62, 1–18.
Woodford, M., 1996. Control of the public debt: A requirement for price stability? NBER WP 5684.
Woodford, M., 2003. Interest and prices. Princeton University Press, Princeton, NJ.
World Economic Outlook, 2005. International Monetary Fund. April 2005.
Yun, T., 1996. Nominal price rigidity, money supply endogeneity, and business cycles. J. Monet. Econ. 37, 345–370.
Yun, T., 2005. Optimal Monetary Policy with Relative Price Distortions. Am. Econ. Rev. 95, 89–109.